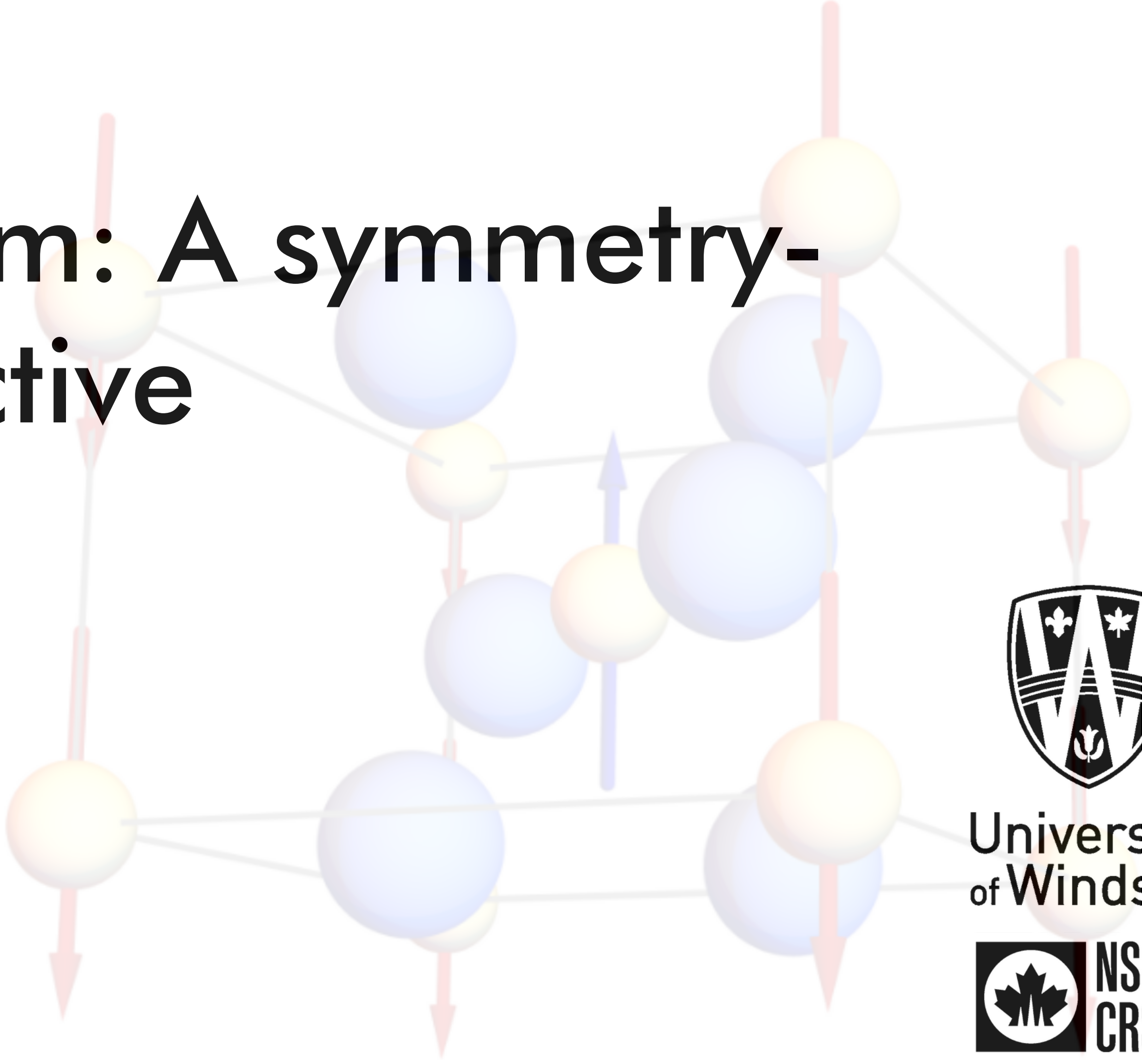
Tutorial:

Altermagnetism: A symmetry-based perspective

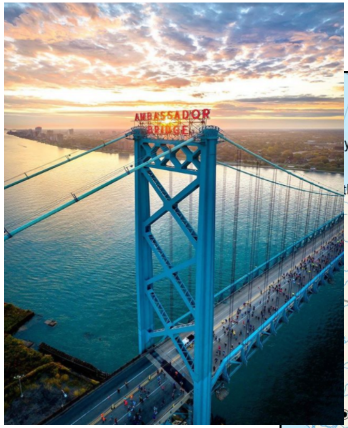
Jeffrey G. Rau

University of Windsor



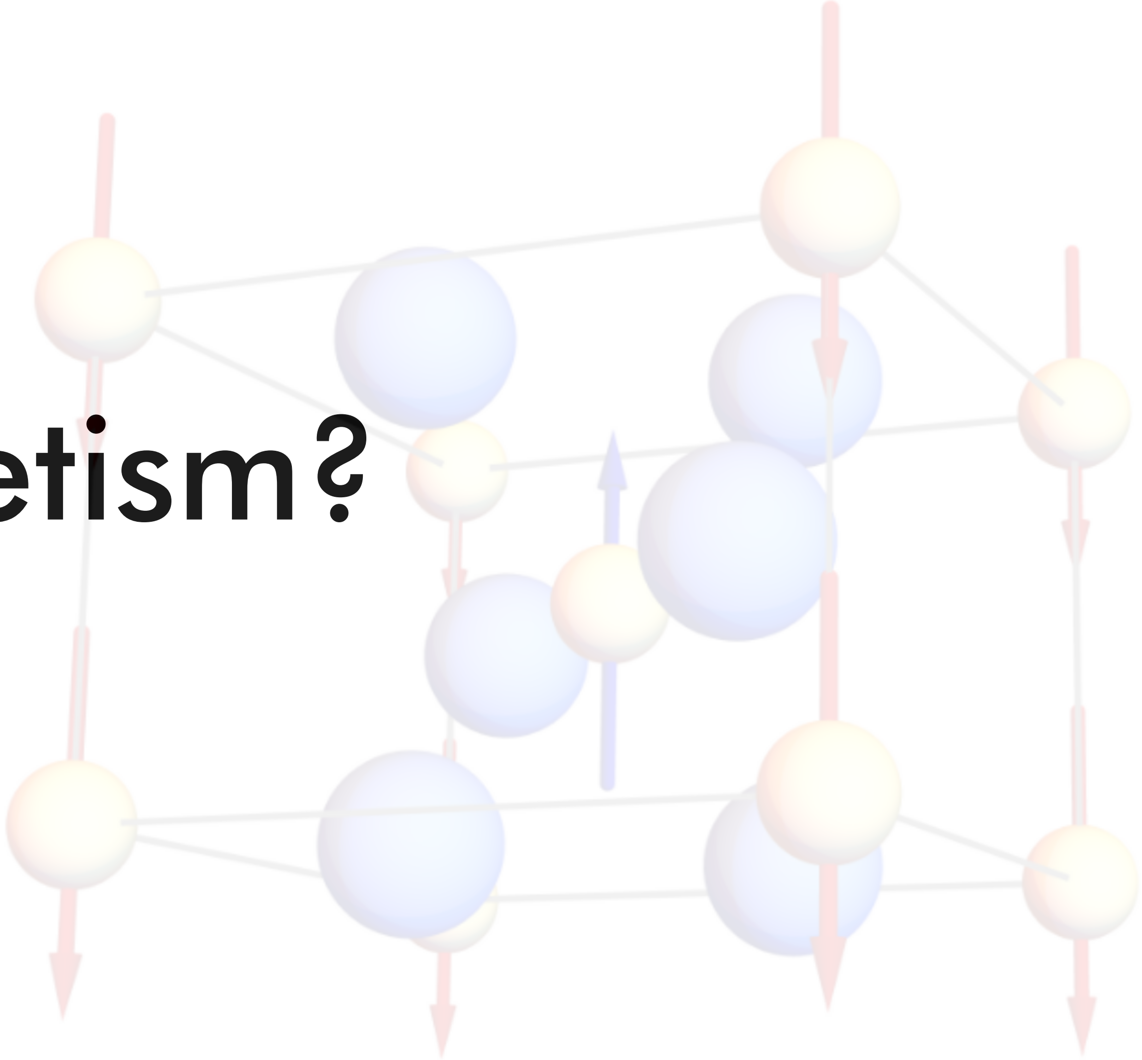
University of Windsor

NSERC CRSNG



Windsor, Ontario, Canada

Altermagnetism?



APS

Altermagnetism Then and Now

Igor Mazin
Physics and Astronomy Department, George Mason University, Fairfax, Virginia

January 8, 2024 • *Physics* 17, 4

Recent theoretical work has identified the possibility of a new and fundamental form of magnetism.

Experimental Evidence for a New Type of Magnetism

January 18, 2024 • *Physics* 17, s10

Spectroscopic data suggest that thin films of a certain semiconducting material can exhibit altermagnetism, a new and fundamental form of magnetism.

Condensed-matter physics

nature

New type of magnetism splits from convention

Carmine Autieri

Magnetic materials with zero net magnetization fall into two classes: conventional antiferromagnets and altermagnets. Physicists have identified a property in altermagnets that widens the divide between the two groups. **See p.517 & p.523**

NEWS | PHYSICS

Researchers discover new kind of magnetism

More than 200 materials could be "altermagnets," predicted just a few years ago

6 FEB 2024 · 3:30 PM ET · BY ZACK SAVITSKY

Science

Science

Scientists Have Confirmed the Existence of a Third Form of Magnetism

This could change the game.

BY CAROLINE DELBERT PUBLISHED: DEC 18, 2024 1:00 PM EST

SAVE ARTICLE

POPULAR

Physics

We’ve discovered a new kind of magnetism. What can we do with it?

Researchers have found the first new type of magnet in nearly a century. Now, these strange "altermagnets" could help us build an entirely new type of computer

By Jacklin Kwan

15 July 2025

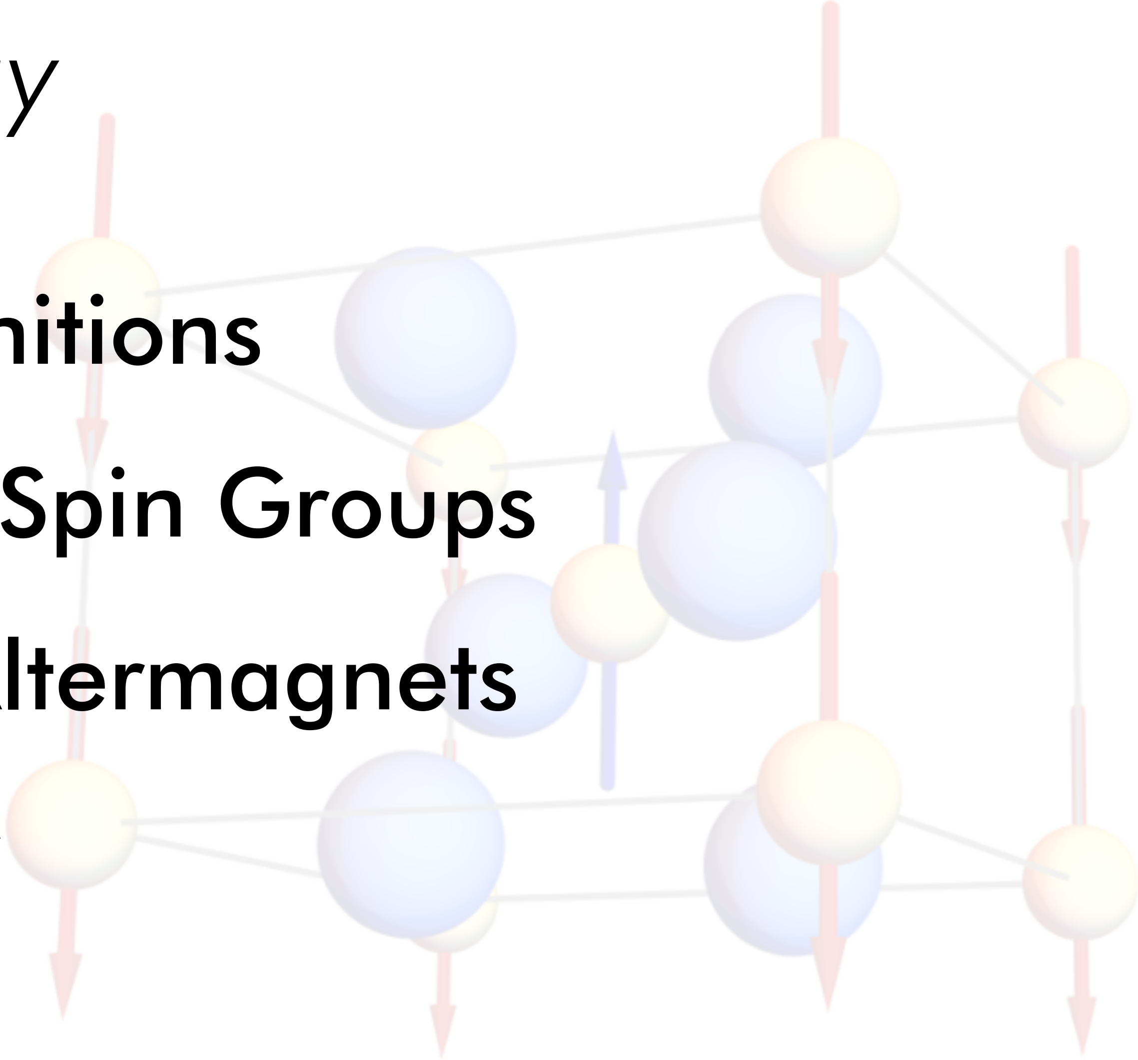
NewScientist

Questions

- Is this really a **third kind of magnetism?**
  - *Why only two? What about spirals and non-colinear or coplanar orders, etc*
- Is this more like a ferromagnet or more like an antiferromagnet?
  - *When is it really distinct from either of those?*
- How does this relate to large body of knowledge on magnetic symmetries?
  - *Like space groups, there are magnetic space groups that have been well-understood for decades*

Outline - *Today*

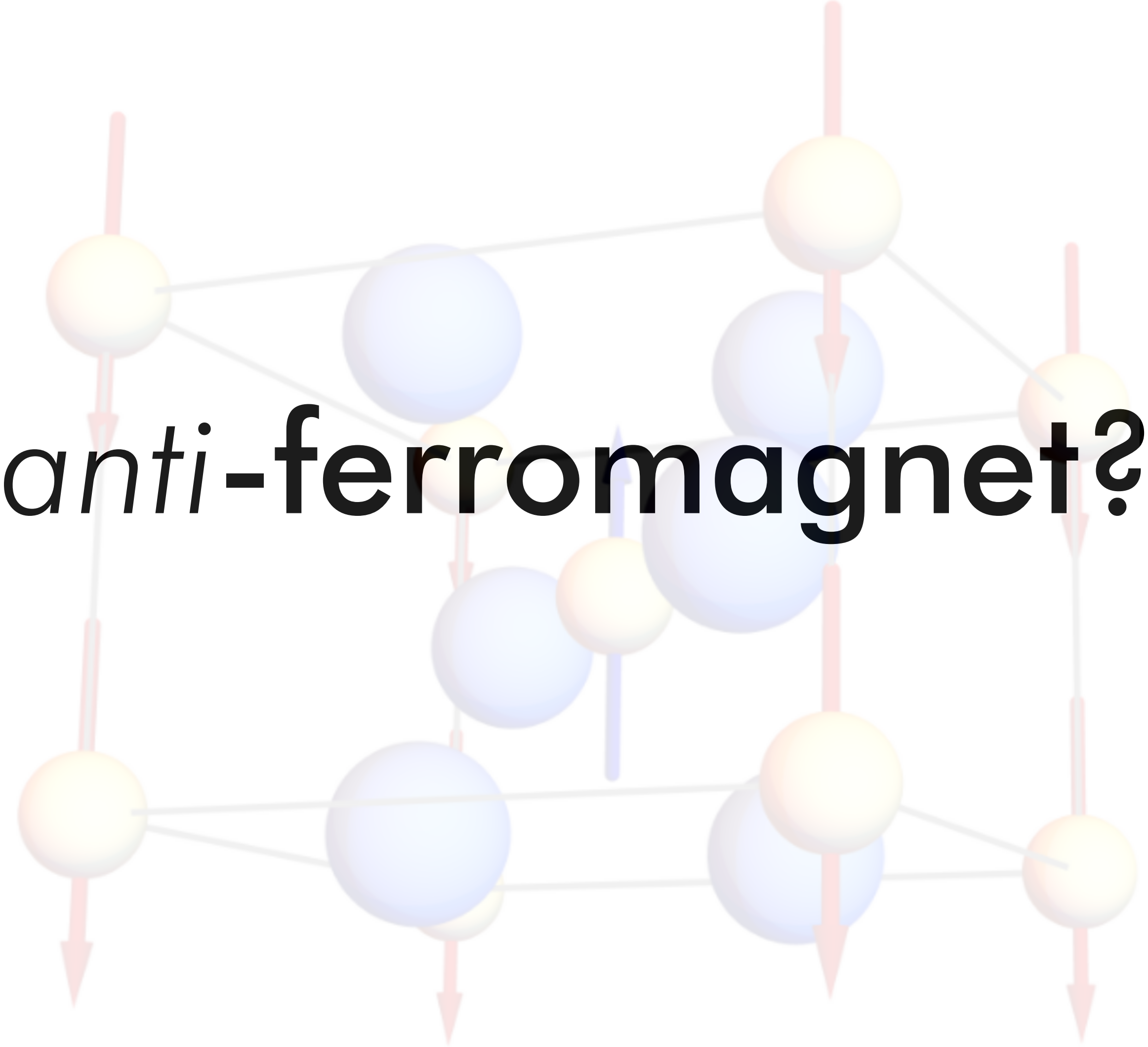
1. History & Definitions
2. Magnetic and Spin Groups
3. Symmetry in Altermagnets
4. Landau theory



Outline - *Tomorrow (tentative)*

1. Insulating altermagnets
2. Altermagnetic magnons
3. Chiral magnon bands
4. Detection, realization & characterization

What is an *anti*-ferromagnet?



1. An interaction that favours the anti-alignment of magnetic moments

2. Two identical but oppositely aligned magnetic sublattices

3. Zero magnetic dipole moment

4. None of the above

ChatGPT 4o

What is an antiferromagnet?

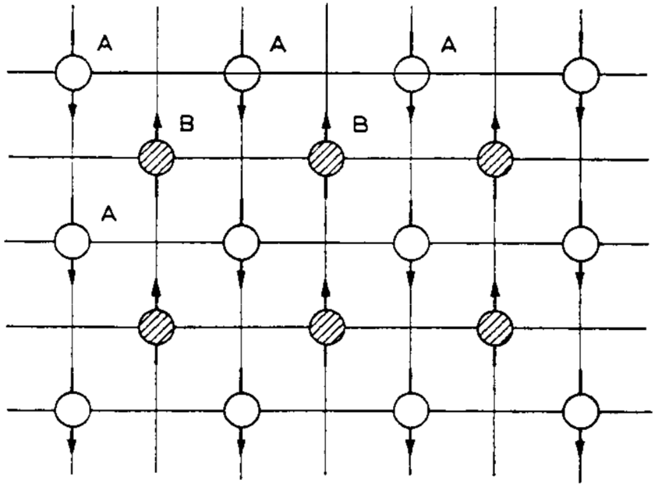
An **antiferromagnet** is a material in which adjacent atomic magnetic moments (spins) align in opposite directions, resulting in **no net macroscopic magnetization** in the absence of an external field. This arrangement minimizes the system's magnetic energy due to **strong exchange interactions** that favor antiparallel spin alignment.

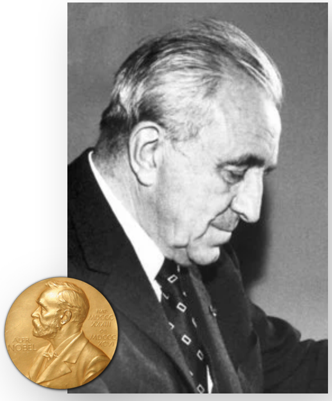
Key Characteristics:

- **Magnetic Ordering**: Below a critical temperature called the **Néel temperature** T_N, the material exhibits long-range magnetic order with alternating spin directions.
- **Zero Net Magnetization**: In contrast to ferromagnets, the opposing magnetic moments cancel out, leading to zero or negligible total magnetization in the bulk.
- **Response to External Fields**: Susceptibility is generally small and decreases as temperature drops below T_N, unlike ferromagnets which show spontaneous magnetization.
- **Examples**: MnO, FeO, NiO, Cr_2O_3, and hematite (Fe_2O_3) under certain conditions.

Still in 1936, I had the idea[5] of applying the theory of the local molecular field to the two sub-lattices A and B with magnetizations J_A and J_B used in the previous papers, and of representing the interactions by imaginary fields H_A and H_B with, at low temperature and for $H=0$, the fundamental relationship $J_B = -J_A$.

The result was that the two sub-lattices had to acquire spontaneous magnetizations in opposite directions, disappearing at a certain transition temperature θ_N, known nowadays as the Néel temperature, following a proposal made by C. J. Gorter. We were hence faced with a new kind of magnetic material, composed of the sum of two interlaced identical ferromagnetics spontaneously magnetized in opposite directions. Effects depending on the square of





LOUIS NÉEL

Magnetism and the local molecular field

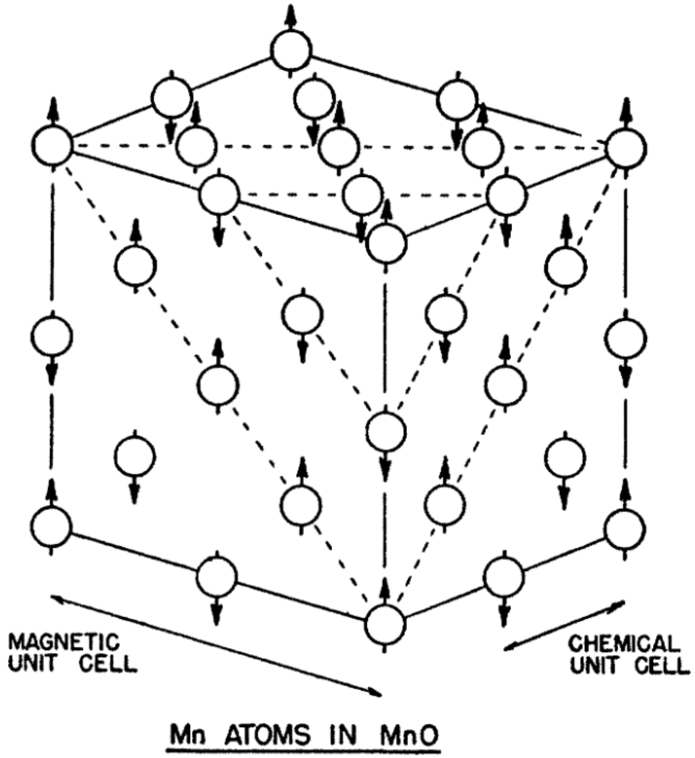
Nobel Lecture, December 11, 1970

Definitions

Assume magnetic order (spontaneous breaking of time-reversal symmetry, smoothly connected to trivial state)

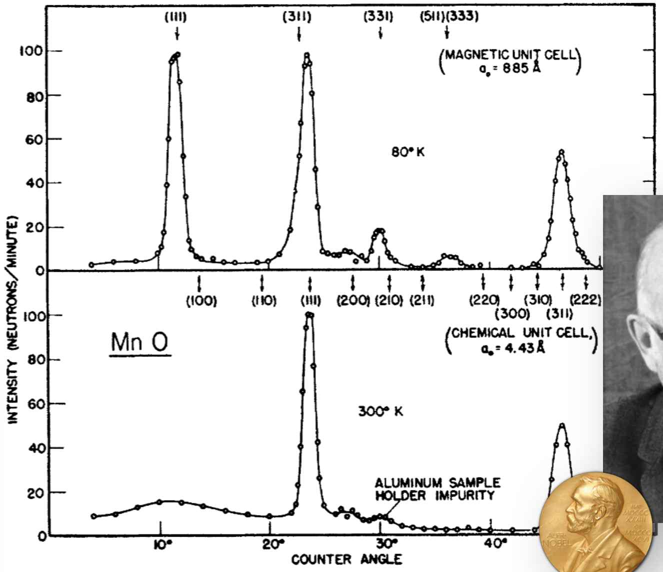
- **Ferromagnetism:** Spontaneous development of a non-zero magnetic dipole moment
- **Antiferromagnetism?** Some options
  - An interaction that favours the anti-alignment of magnetic moments ← *Tendency to anti-align*
  - Two *identical* but oppositely aligned magnetic sublattices* ← *Néel's definition*
  - Zero magnetic dipole moment ← *"Modern" definition*

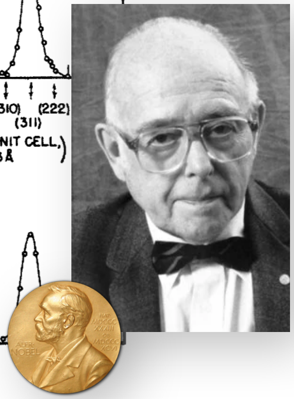
**Néel's definition implies the modern one, but not vice-versa*

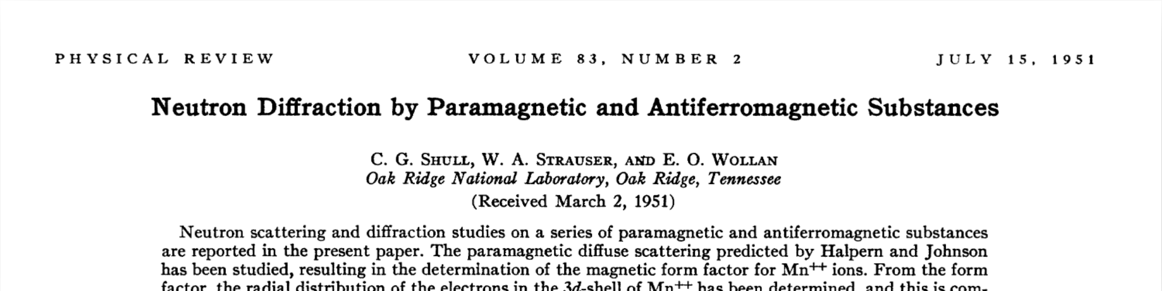


Manganese Oxide (MnO)

- Antiferromagnetic interactions? ✅
- Identical, anti-aligned sublattices? ✅
- Zero net moment? ✅







PHYSICAL REVIEW VOLUME 83, NUMBER 2 JULY 15, 1951

Neutron Diffraction by Paramagnetic and Antiferromagnetic Substances

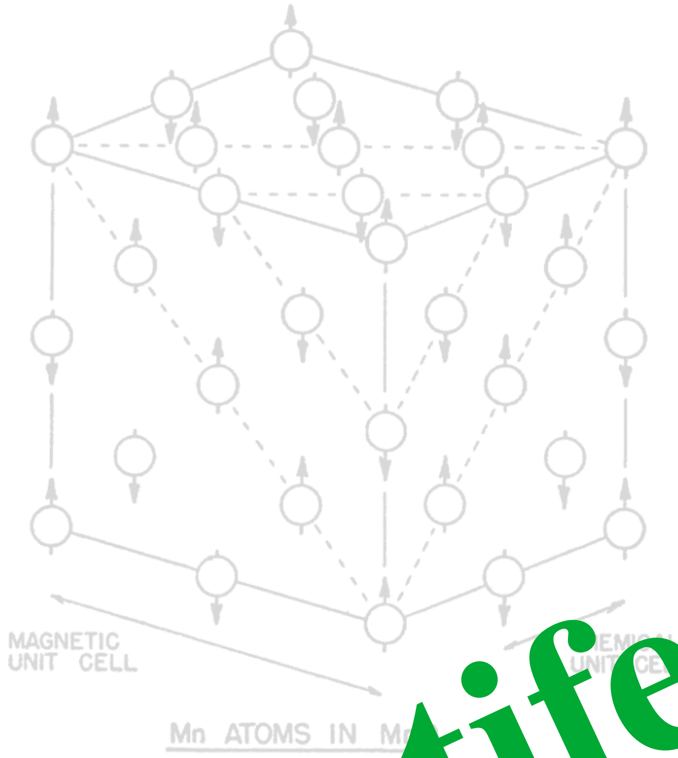
C. G. Shull, W. A. Strauser, and E. O. Wollan
Oak Ridge National Laboratory, Oak Ridge, Tennessee
(Received March 2, 1951)

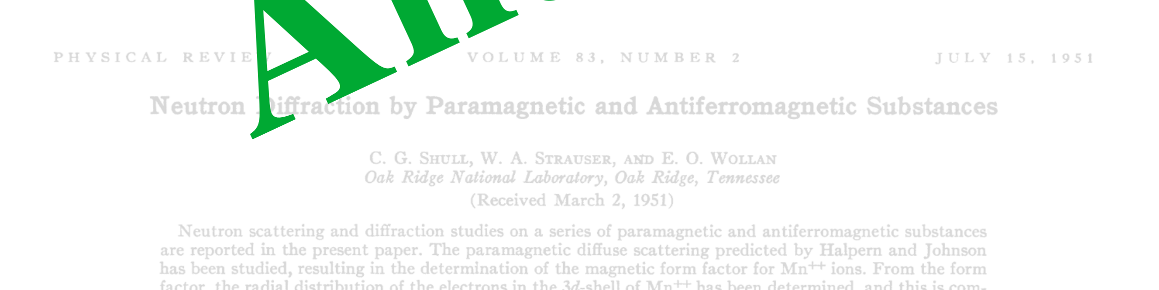
Neutron scattering and diffraction studies on a series of paramagnetic and antiferromagnetic substances are reported in the present paper. The paramagnetic diffuse scattering predicted by Halpern and Johnson has been studied, resulting in the determination of the magnetic form factor for Mn^{++} ions. From the form factor, the radial distribution of the electrons in the 3*d*-shell of Mn^{++} has been determined, and this is com-

Manganese Oxide (MnO)

- Antiferromagnetic interactions? ✅
- Identical, anti-aligned sublattices? ✅
- Zero net moment?

Antiferromagnet





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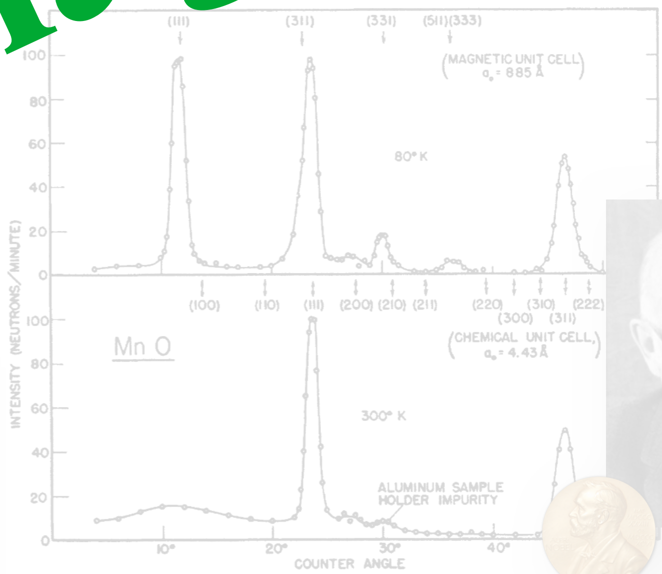
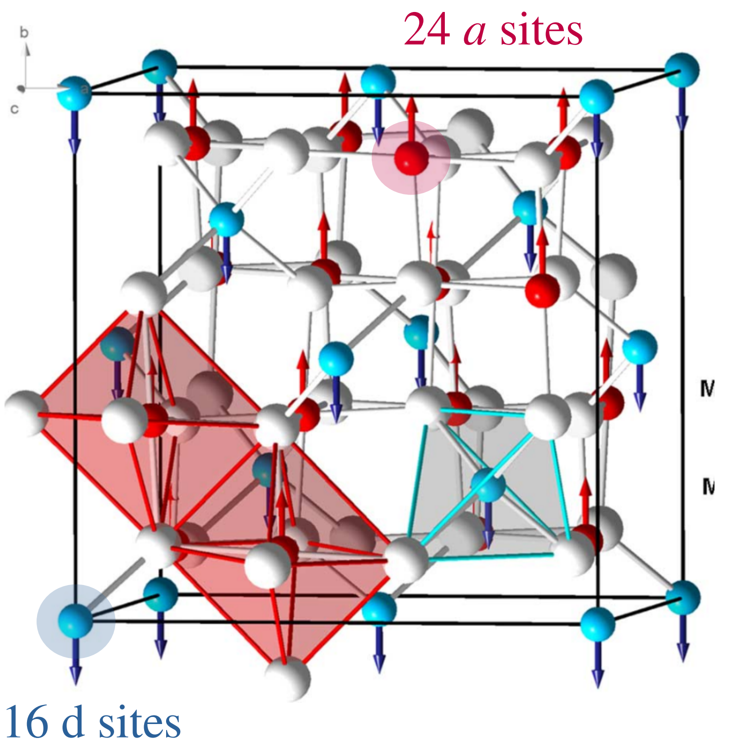
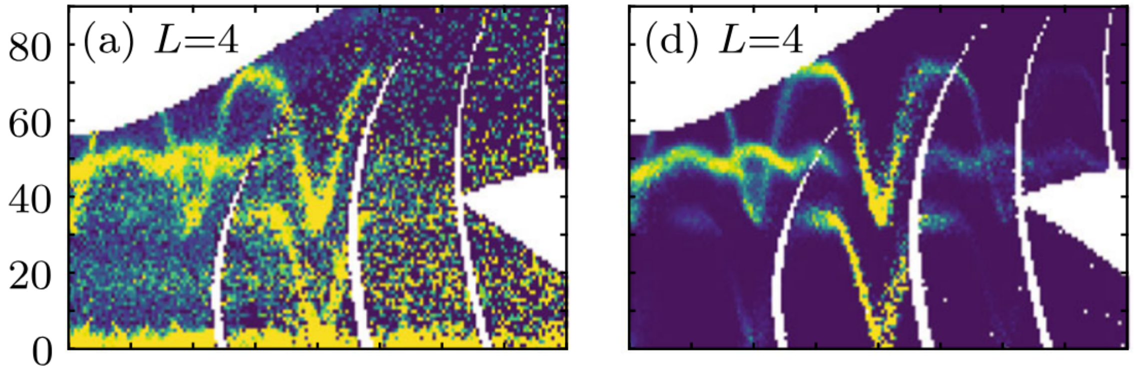


Yttrium Iron Garnet
"YIG" ($Y_3Fe_5O_{12}$)

"the fruitfly of magnetism" (C. Kittel)





	This work (meV)
J_1	6.8(2)
J_2	0.52(4)
J_{3a}	0.0(1)
J_{3b}	1.1(3)

Net magnetic moment

~ 40 μ_B / unit cell

- Antiferromagnetic interactions? ✅
- Identical, anti-aligned sublattices? ❌
- Zero net moment? ❌

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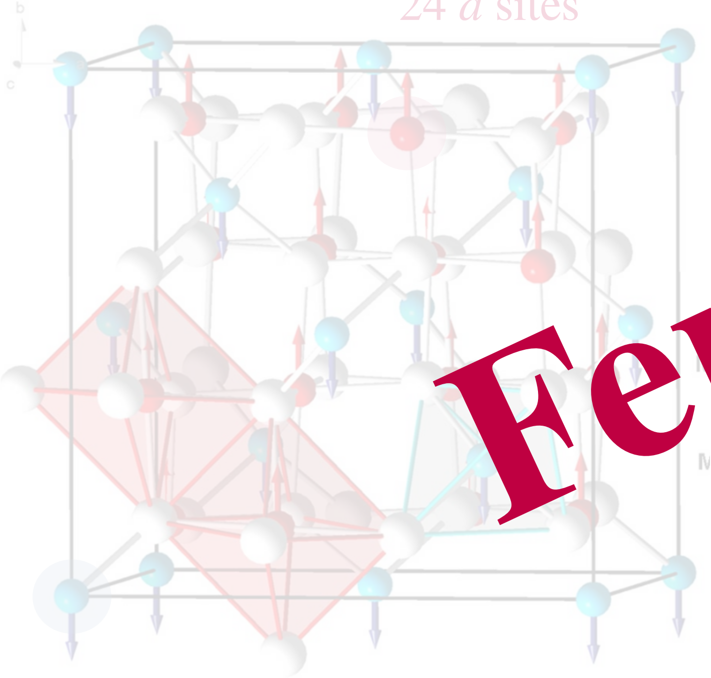

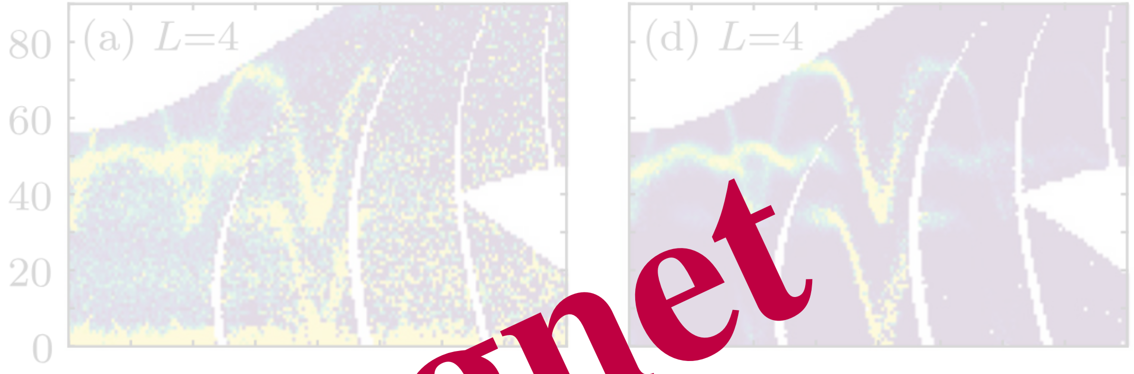

Net magnetic moment

~[illegible]0 μ_B / unit cell

	This work (meV)
J_1	6.8(2)
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- Antiferromagnetic interactions? ✅
- Identical, anti-aligned sublattices? ❌
- Zero net moment? ❌

Ferromagnet

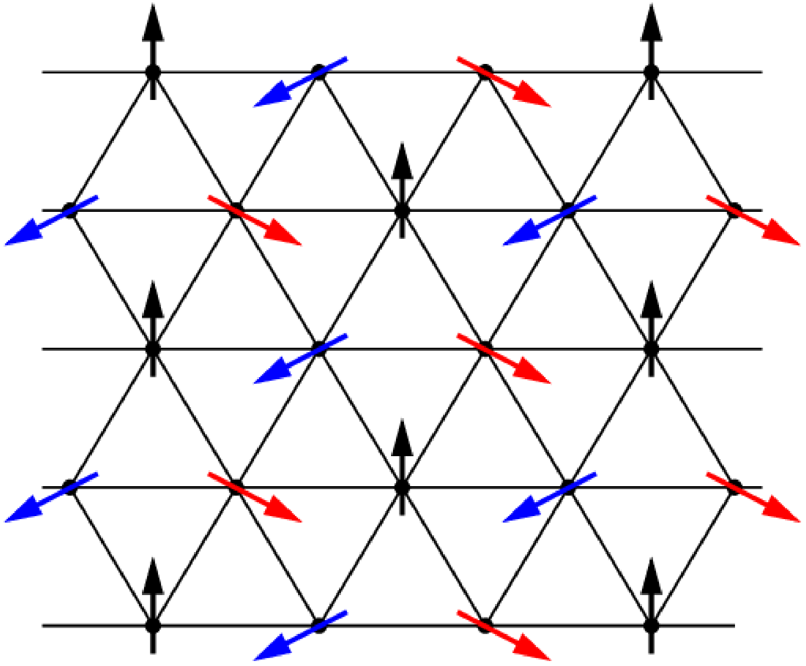
(Ferrimagnet)

Triangular HAFM

- Ground state of antiferromagnetic Heisenberg model on the triangular lattice

$$J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

- *Frustrated* – can't all anti-align
  - Compromise: Three sublattice 120° order
- **Compensated** (no net moment)
  - Sublattices cancel



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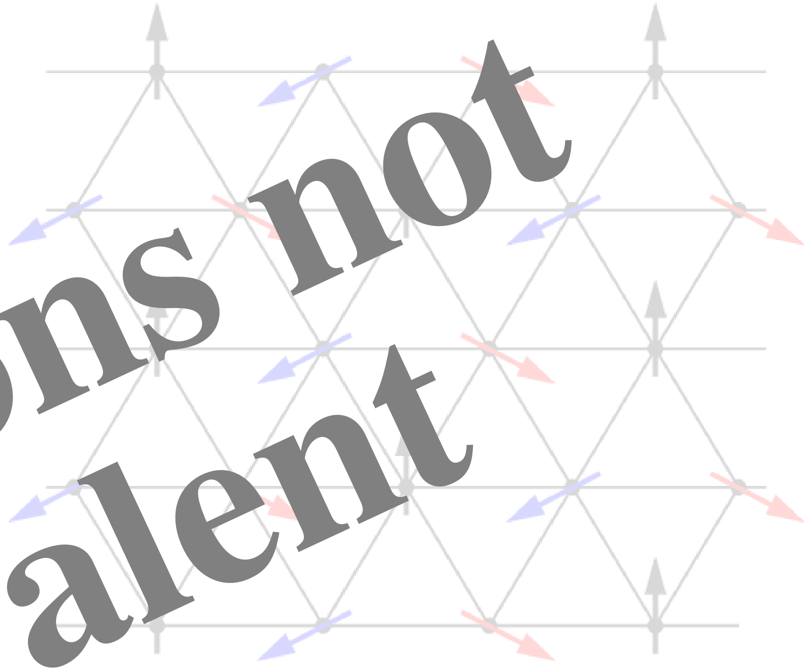
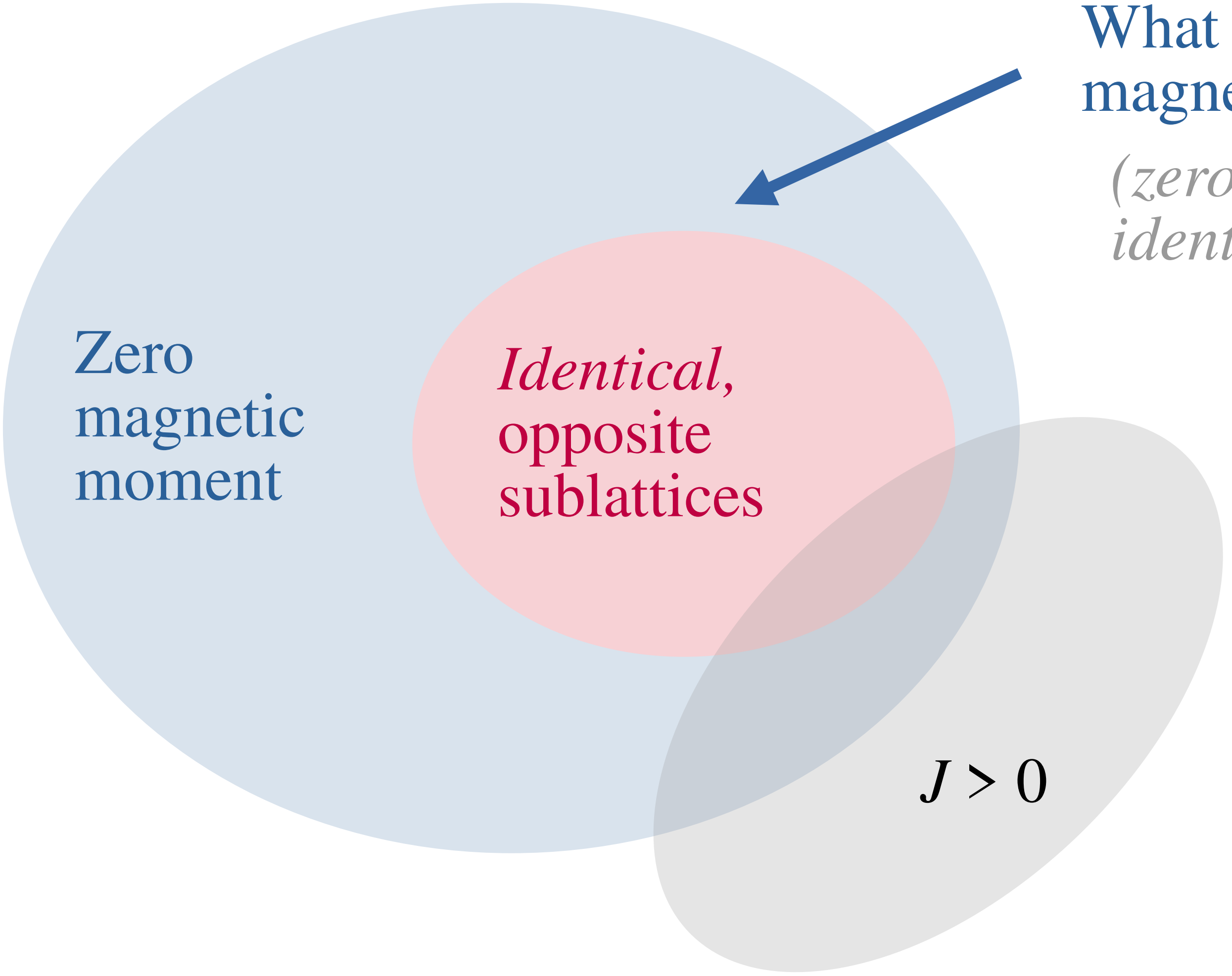


- Antiferromagnetic interactions? ✅
- Identical, anti-aligned sublattices? ❌
- Zero net moment? ✅

Definitions not equivalent

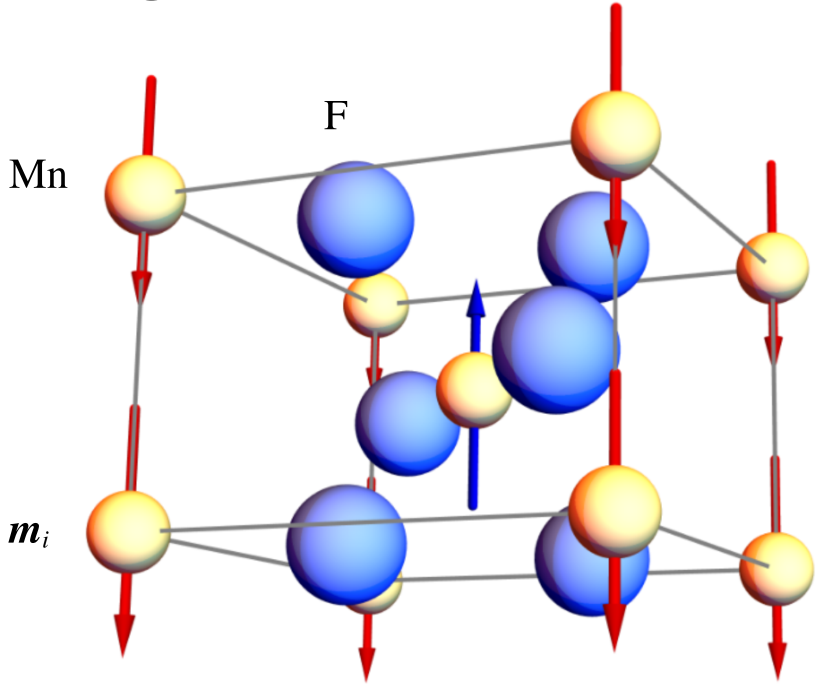


What kind of interesting magnetic structures live *here?*

(zero net moment, but not identical sublattices)

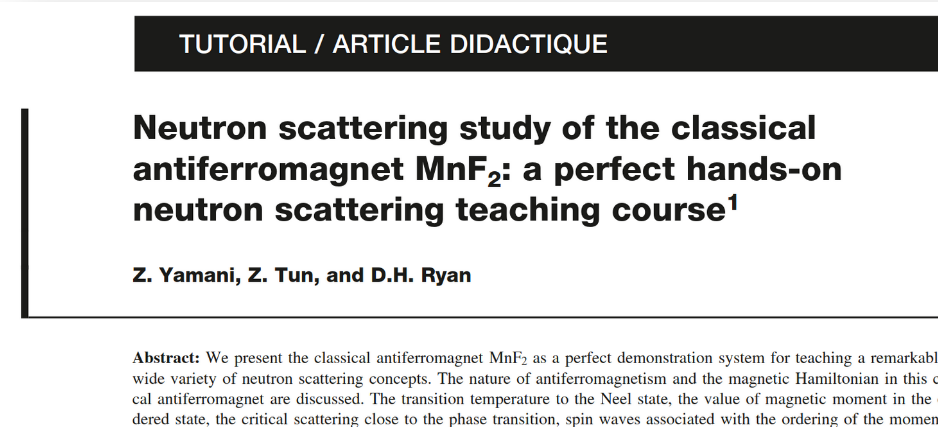
What do we actually mean by *identical*?

Manganese Diflouride (MnF_2)



- Insulating $S = 5/2$ magnet on a tetragonal lattice
- AF exchange, nearly classical
- Two-sublattice order, oppositely aligned
- Well-studied for 60+ years

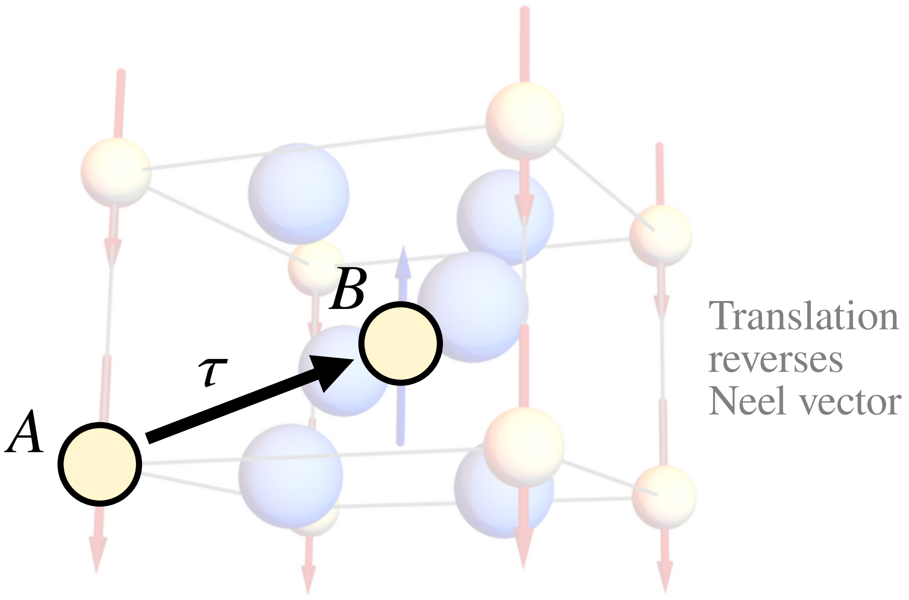
- Antiferromagnetic interactions? ✅
- *Identical*, anti-aligned sublattices? ❓
- Zero net moment? ✅

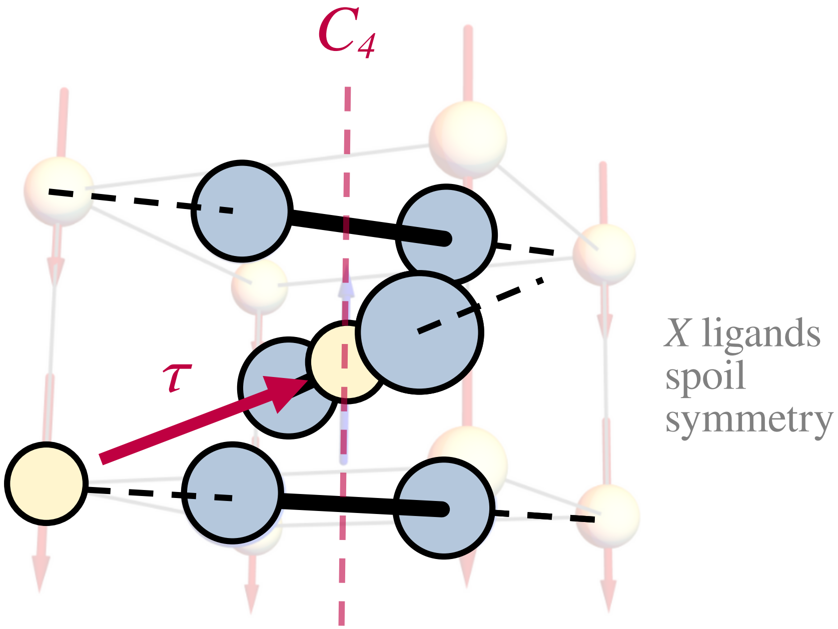

TUTORIAL / ARTICLE DIDACTIQUE

Neutron scattering study of the classical antiferromagnet MnF_2: a perfect hands-on neutron scattering teaching course[1]

Z. Yamani, Z. Tun, and D.H. Ryan

Abstract: We present the classical antiferromagnet MnF_2 as a perfect demonstration system for teaching a remarkabl
wide variety of neutron scattering concepts. The nature of antiferromagnetism and the magnetic Hamiltonian in this c
cal antiferromagnet are discussed. The transition temperature to the Neel state, the value of magnetic moment in the
dered state, the critical scattering close to the phase transition, spin waves associated with the ordering of the momen





Translation connecting *A* and *B* sublattice *not a symmetry* ...

… need to follow with a four-fold rotation about *z*-axis

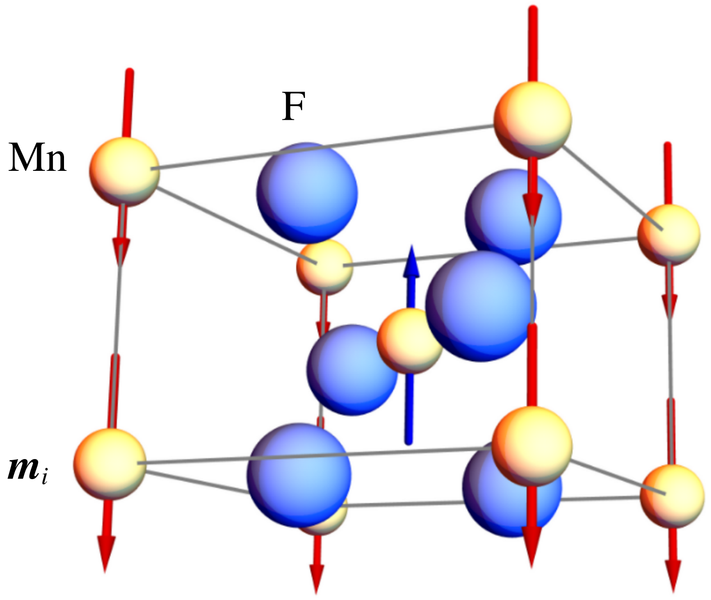
… magnetic dipole moments are identical, but crystal *environment* is not

Definition of an *alter*-magnet

- Colinear antiferromagnetic arrangement of magnetic moments

Not exactly "identical"

- Opposite sublattices *symmetry-related*
- **… but by a *non-trivial spatial symmetry* (rotation or reflection)**
  - *Not* translation
  - *Not* inversion
- Can be defined with or without spin-rotation symmetry, in metal or insulator



Moment structure in MnF_2

See: Yuan *et al*, Phys Rev. B **102** 014422 (2020), Šmejkal *et al* Phys. Rev. X **12**, 031042 (2022), Šmejkal *et al*, Phys. Rev. X **12**, 040501 (2022), Mazin, Phys. Rev. X **12** 040002 (2022), … and many more ...

More signatures of altermagnetism

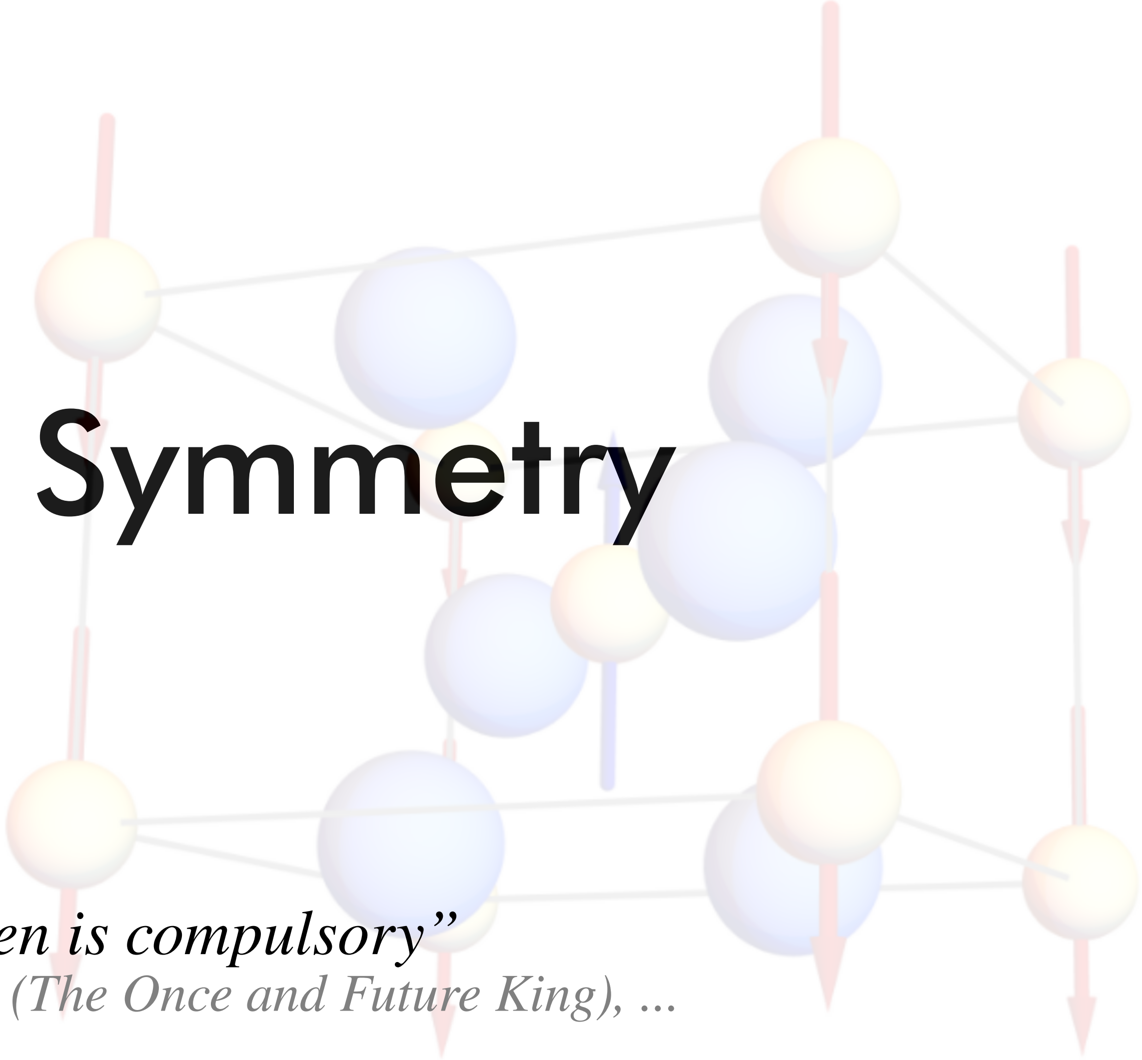
- Think of as antiferromagnets – **no net moment** - but with many properties *usually* associated with *ferromagnets* *No stray fields*
- Features that are ferromagnetic-like
  - Spin-split bands
  - Anomalous Hall Effect
  - Magneto-optical response
  - Piezomagnetism
  - Spin-polarized currents, spin-transfer torque, ...

Applications in Spintronics?

- **New features?** Induced multipolar order?

Yuan *et al*, Phys Rev. B **102** 014422 (2020), Šmejkal *et al.* Phys. Rev. X **12**, 031042 (2022); Šmejkal *et al*, Science Advances **6**, eaaz8809 (2020), Bhowal & Spaldin, Phys. Rev. X 14, 011019 (2024), …

Magnetic Symmetry



"Everything not forbidden is compulsory"

- M. Gellman, T. H. White (The Once and Future King), ...

Crystal Symmetries

- Symmetries are combinations of *rotations, inversions & translations*

$$\{\pm \boldsymbol{R}|\boldsymbol{\tau}\}$$

Rotation part, without or without inversion

Translation part

- **Space group:** All such operations that leave the crystal invariant
  - *Point group:* Space group with translations modded out

Acts as expected on position $\boldsymbol{r}$

$$\{\pm \boldsymbol{R}|\boldsymbol{\tau}\}\boldsymbol{r} = \pm \boldsymbol{R}\boldsymbol{r} + \boldsymbol{\tau}$$

Pseudo-vector invariant under inversion

$$\{\pm \boldsymbol{R}|\boldsymbol{\tau}\}[\boldsymbol{m}(\boldsymbol{r})] = \boldsymbol{R}\boldsymbol{m}(\pm \boldsymbol{R}^{-1}(\boldsymbol{r} - \boldsymbol{\tau}))$$

Pseudo-vector field $\boldsymbol{m}(\boldsymbol{r})$

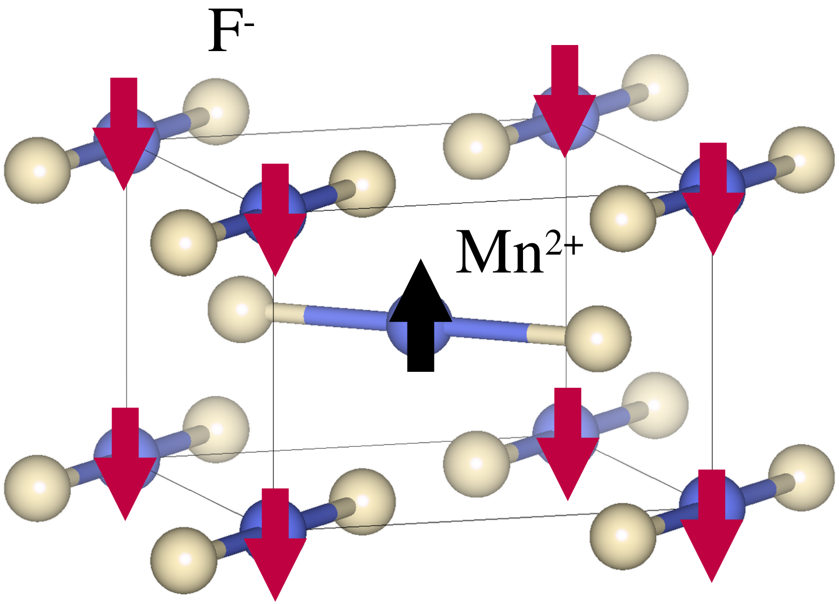
$I =$ inversion

$C_{n,\hat{\boldsymbol{v}}} =$ $2\pi/n$ rotation along $\boldsymbol{v}$

$\sigma_{\hat{\boldsymbol{v}}} =$ reflect in plane $\perp$ $\boldsymbol{v}$

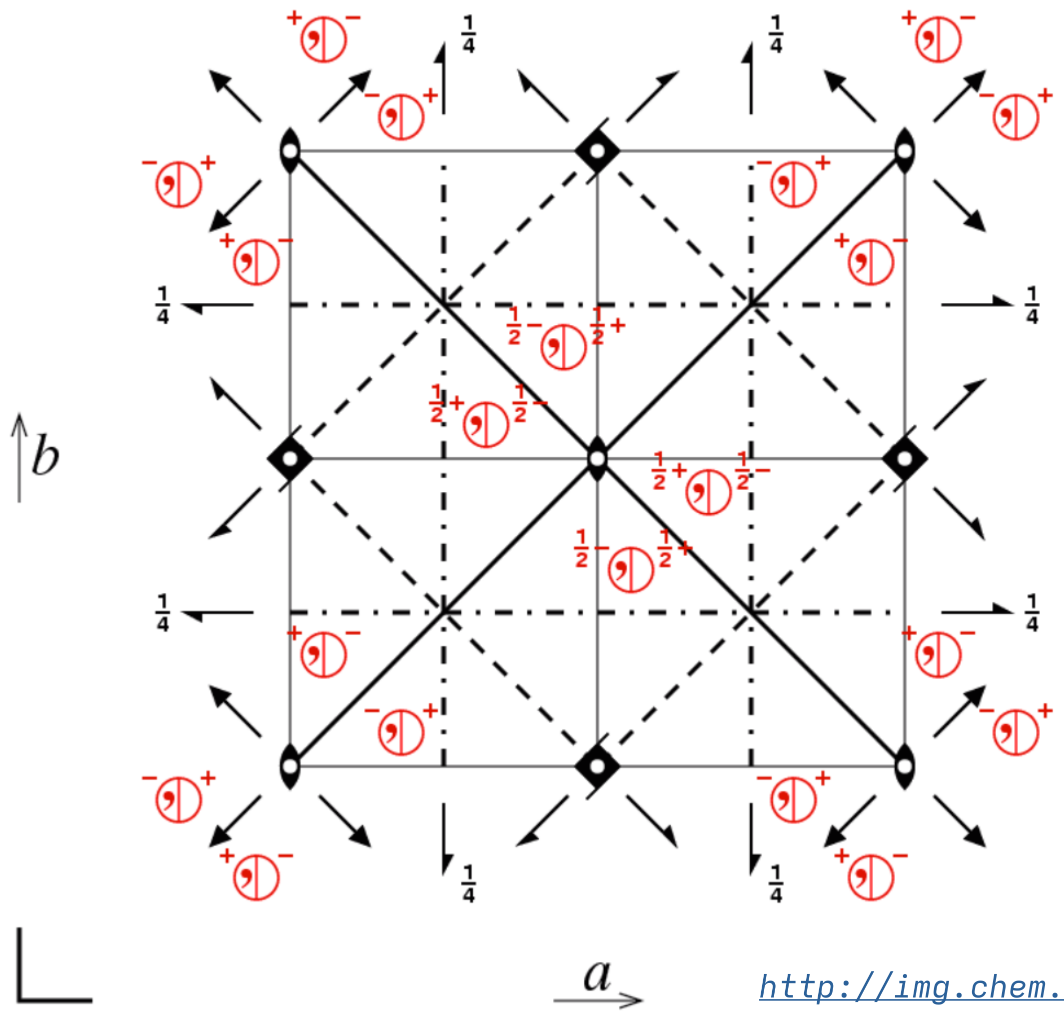
Example: MnF_2

- Space group of *crystal* is $P4_2/mnm$ (#136)
- Unit cell contains:
  - Two Mn^{2+} ions (magnetic)
  - Four F^- ions (non-magnetic)
- Magnetic ordering is *within* the unit cell
  - Propagation vector is $\boldsymbol{k} = \mathbf{0}$
- Experimentally $\boldsymbol{m}_i = \boldsymbol{m}(\boldsymbol{r}_i)$ at each site along z



$$\boldsymbol{m}_i = \pm m\hat{\boldsymbol{z}}$$

$P4_2/mnm$ $\quad P\,4_2/m\;2_1/n\;2/m$ $\quad 4/mmm$ $\quad$ No. 136

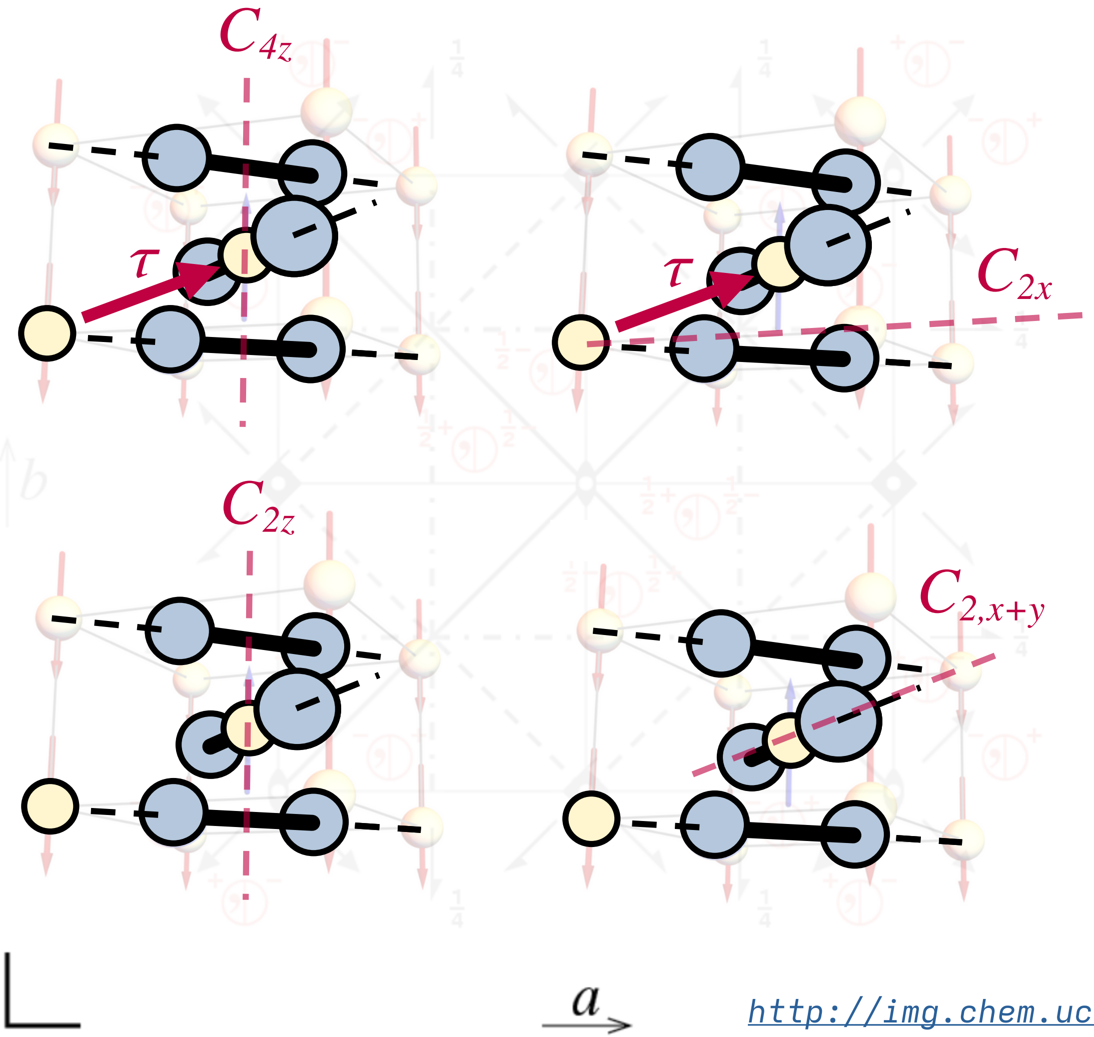


Id.

No.	Coordinates	Element
1	x, y, z	
2	$\bar{x}, \bar{y}, z$	C_{2z}
3	$\frac{1}{2}-y, \frac{1}{2}+x, \frac{1}{2}+z$	$C_{4z}+\tau, C_{4z}^{-1}+\tau$
4	$\frac{1}{2}+y, \frac{1}{2}-x, \frac{1}{2}+z$	
5	$\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z$	$C_{2x}+\tau, C_{2y}+\tau$
6	$\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}+z$	
7	y, x, z	$C_{2,x+y}, C_{2,x-y}$
8	$\bar{y}, \bar{x}, z$	
9	$\bar{x}, \bar{y}, \bar{z}$	… same elements + inversion
10	$x, y, \bar{z}$	
11	$\frac{1}{2}+y, \frac{1}{2}-x, \frac{1}{2}-z$	
12	$\frac{1}{2}-y, \frac{1}{2}+x, \frac{1}{2}-z$	
13	$\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}-z$	… + all translations
14	$\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}-z$	
15	$\bar{y}, \bar{x}, \bar{z}$	
16	$y, x, \bar{z}$	

http://img.chem.ucl.ac.uk/sgp/

$P4_2/mnm$ $P\,4_2/m\,2_1/n\,2/m$ $4/mmm$ No. 136



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13	$\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}-z$	
14	$\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}-z$	… + all translations
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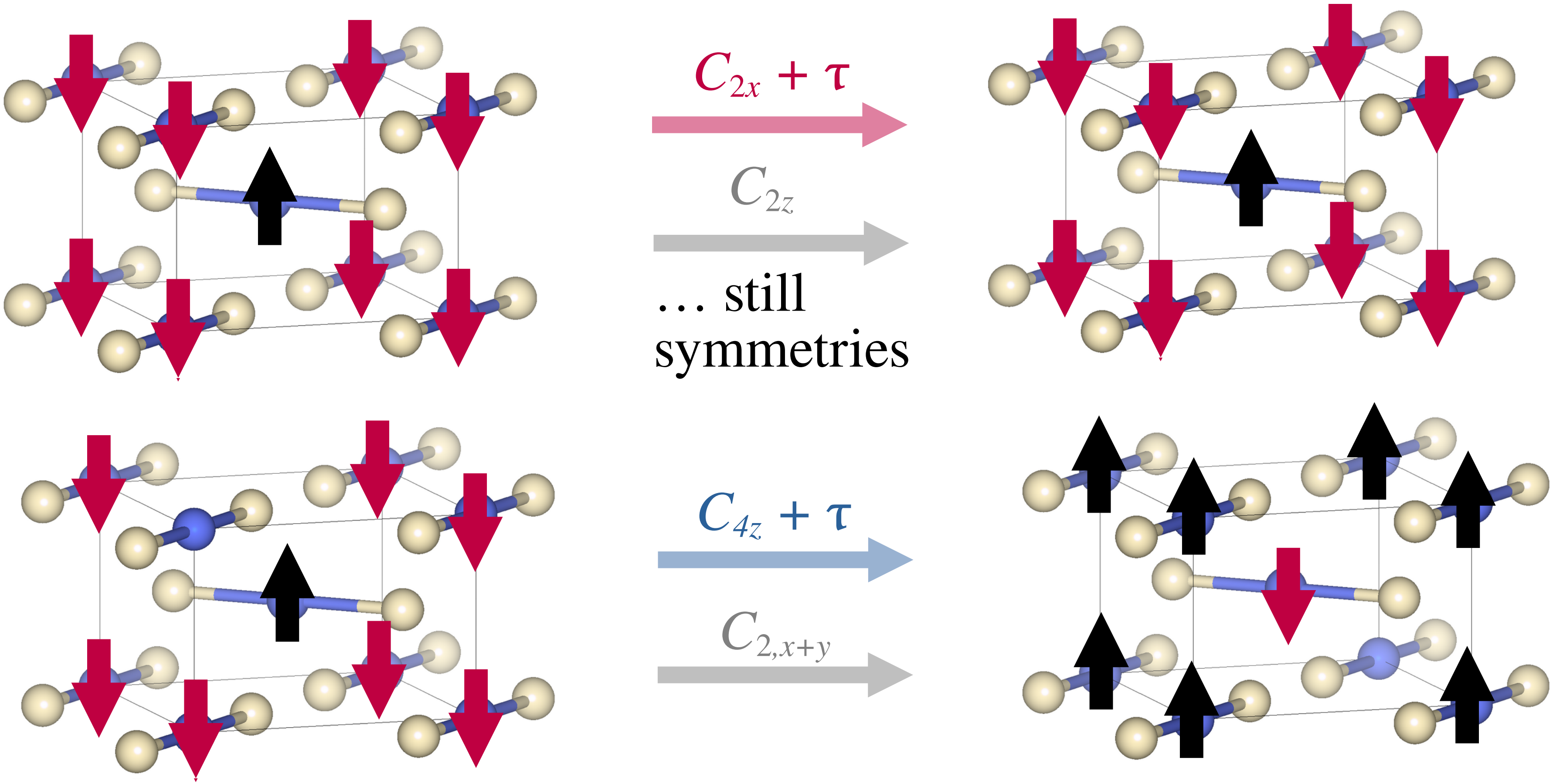
16	$y, x, \bar{z}$	

http://img.chem.ucl.ac.uk/sgp/

What about symmetries of *magnetic structure*?



Symmetry when combined with **time reversal**

- Space group promoted to **magnetic space group**
  - Subset of elements paired with *time-reversal*
- **MSG:** *$P4_2'/mnm'$* (#136.499)

C_{2z}

$C_{4z} + \tau, C_{4z}^{-1} + \tau$

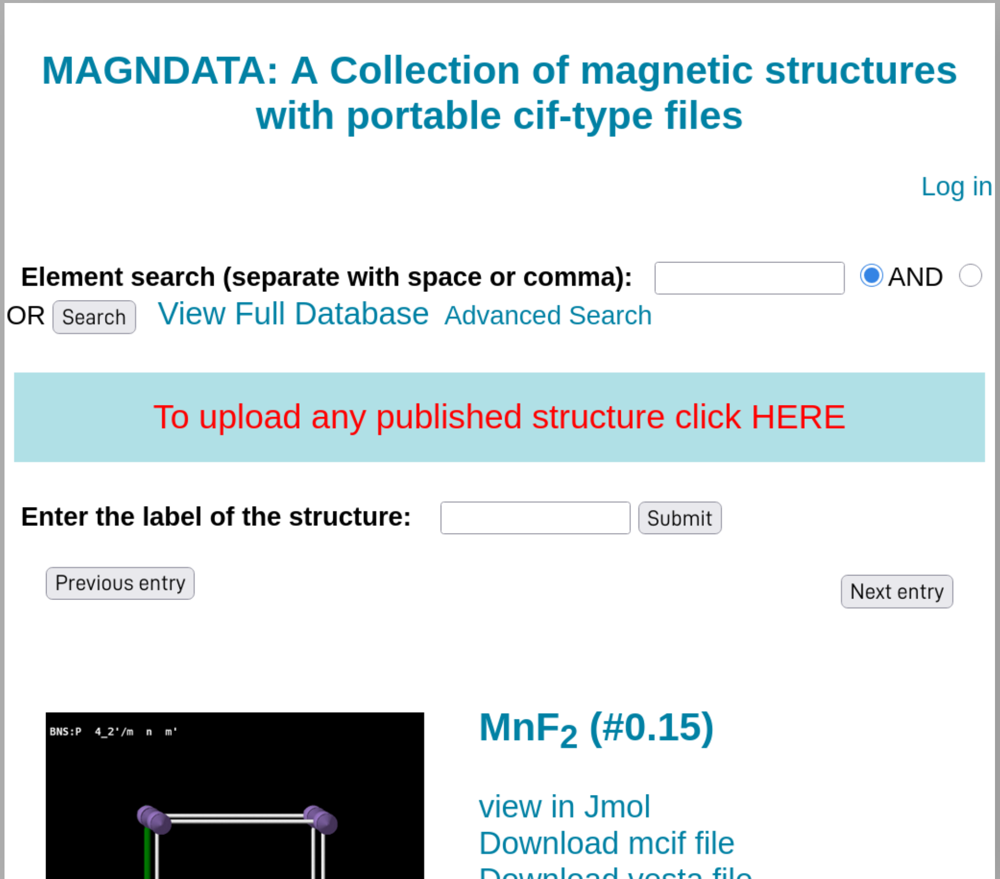
$C_{2x} + \tau, C_{2y} + \tau$

$C_{2,x+y}, C_{2,x-y}$

Anything involving these comes with T.R.

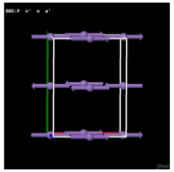
*Easy to find on **Bilbao Crystallographic Server (MAGNDATA)***

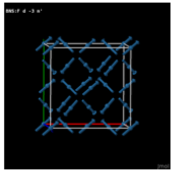
https://www.cryst.ehu.es/magndata/

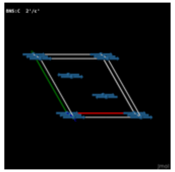


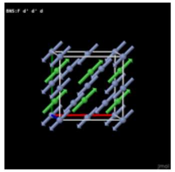
Aside

Zero propagation vector


0.1 $LaMnO_3$


0.2 $Cd_2Os_2O_7$

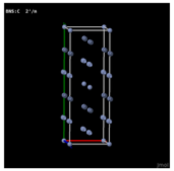

0.3 Ca_3LiOsO_6

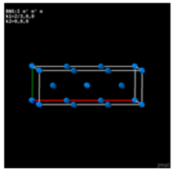

0.4 $NiCr_2O_4$

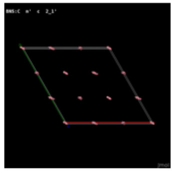

0.5 Cr_2S_3

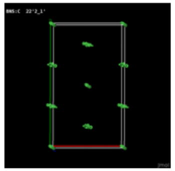
Click to expand/shrink back the rest

Non-zero propagation vector (magnetic space groups of Type I or Type III)


1.0.1 Ag_2CrO_2


1.0.2 $URu_{0.96}Rh_{0.04}Si_2$

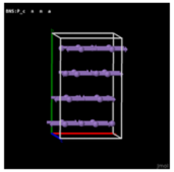

1.0.3 $CsCoBr_3$

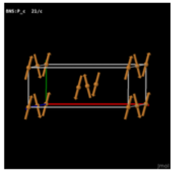

1.0.4 $CsNiCl_3$

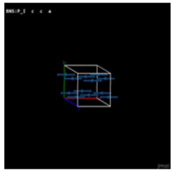

1.0.5 Sr_3CoIrO_6

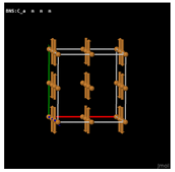
Click to expand/shrink back the rest

Non-zero propagation vector (magnetic space groups of Type IV)


1.1 Mn_3O_4


1.2 $CuSe_2O_5$


1.3 Sr_2IrO_4

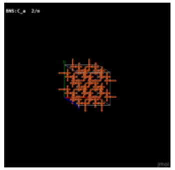

1.4 $YBa_2Cu_3O_{6+d}$

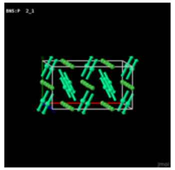

1.5 $YBa_2Cu_3O_{6+d}$

Click to expand/shrink back the rest

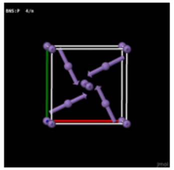
Two propagation vectors


2.1 $EuFe_2As_2$

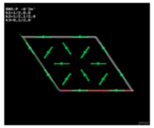

2.2 $Sr_2F_2Fe_2OS_2$

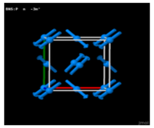

2.3 $HoNiO_3$

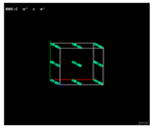

2.4 $Eu(Fe_{0.82}Co_{0.18})As_2$

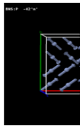

2.5 Mn_3CuN

Three or more propagation vectors


3.1 TmAgGe


3.2 UO_2


3.3 Ho_2RhIn_8


3.4 M

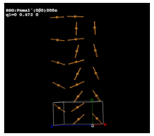
Click to expand/shrink back the rest

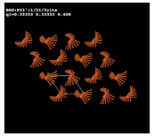
INCOMMENSURATE STRUCTURES

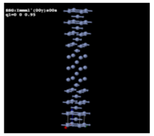
Click to expand/shrink back the rest

INCOMMENSURATE STRUCTURES

One propagation vector


1.1.1 Cs_2CuCl_4


1.1.2 $RbFe(MoO_4)_2$


1.1.3 Cr


1.1.4

Click to expand/shrink back the rest

N	Standard/Default Setting			
	(x,y,z) form	Matrix form	Geom. interp.	Seitz notation
1	x, y, z, +1 m_x,m_y,m_z	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	1 +1	$\{ 1 \mid 0 \}$
2	x+1/2, -y+1/2, -z+1/2, +1 $m_x,-m_y,-m_z$	$\begin{pmatrix} 1 & 0 & 0 & 1/2 \\ 0 & -1 & 0 & 1/2 \\ 0 & 0 & -1 & 1/2 \end{pmatrix}$	2 (1/2,0,0) x,1/4,1/4 +1	$\{ 2_{100} \mid 1/2\ 1/2\ 1/2 \}$
3	-x+1/2, y+1/2, -z+1/2, +1 $-m_x,m_y,-m_z$	$\begin{pmatrix} -1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & -1 & 1/2 \end{pmatrix}$	2 (0,1/2,0) 1/4,y,1/4 +1	$\{ 2_{010} \mid 1/2\ 1/2\ 1/2 \}$
4	-x, -y, z, +1 $-m_x,-m_y,m_z$	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	2 0,0,z +1	$\{ 2_{001} \mid 0 \}$
5	-x, -y, -z, +1 m_x,m_y,m_z	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	-1 0,0,0 +1	$\{ -1 \mid 0 \}$
6	-x+1/2, y+1/2, z+1/2, +1 $m_x,-m_y,-m_z$	$\begin{pmatrix} -1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}$	n (0,1/2,1/2) 1/4,y,z +1	$\{ m_{100} \mid 1/2\ 1/2\ 1/2 \}$
7	x+1/2, -y+1/2, z+1/2, +1 $-m_x,m_y,-m_z$	$\begin{pmatrix} 1 & 0 & 0 & 1/2 \\ 0 & -1 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}$	n (1/2,0,1/2) x,1/4,z +1	$\{ m_{010} \mid 1/2\ 1/2\ 1/2 \}$
8	x, y, -z, +1 $-m_x,-m_y,m_z$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	m x,y,0 +1	$\{ m_{001} \mid 0 \}$

9	-y+1/2, x+1/2, z+1/2, -1 $m_y,-m_x,-m_z$	$\begin{pmatrix} 0 & -1 & 0 & 1/2 \\ 1 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}'$	4^+ (0,0,1/2) 0,1/2,z -1	$\{ 4'^{+}_{001} \mid 1/2\ 1/2\ 1/2 \}$
10	y+1/2, -x+1/2, z+1/2, -1 $-m_y,m_x,-m_z$	$\begin{pmatrix} 0 & 1 & 0 & 1/2 \\ -1 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}'$	4^- (0,0,1/2) 1/2,0,z -1	$\{ 4'^{-}_{001} \mid 1/2\ 1/2\ 1/2 \}$
11	y, x, -z, -1 $-m_y,-m_x,m_z$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}'$	2 x,x,0 -1	$\{ 2'_{110} \mid 0 \}$
12	-y, -x, -z, -1 m_y,m_x,m_z	$\begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}'$	2 x,-x,0 -1	$\{ 2'_{1-10} \mid 0 \}$
13	y+1/2, -x+1/2, -z+1/2, -1 $m_y,-m_x,-m_z$	$\begin{pmatrix} 0 & 1 & 0 & 1/2 \\ -1 & 0 & 0 & 1/2 \\ 0 & 0 & -1 & 1/2 \end{pmatrix}'$	-4^+ 1/2,0,z 1/2,0,1/4 -1	$\{ -4'^{+}_{001} \mid 1/2\ 1/2\ 1/2 \}$
14	-y+1/2, x+1/2, -z+1/2, -1 $-m_y,m_x,-m_z$	$\begin{pmatrix} 0 & -1 & 0 & 1/2 \\ 1 & 0 & 0 & 1/2 \\ 0 & 0 & -1 & 1/2 \end{pmatrix}'$	-4^- 0,1/2,z 0,1/2,1/4 -1	$\{ -4'^{-}_{001} \mid 1/2\ 1/2\ 1/2 \}$
15	-y, -x, z, -1 $-m_y,-m_x,m_z$	$\begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}'$	m x,-x,z -1	$\{ m'_{110} \mid 0 \}$
16	y, x, z, -1 m_y,m_x,m_z	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}'$	m x,x,z -1	$\{ m'_{1-10} \mid 0 \}$

… still symmetries

Symmetry when combined with **time reversal**

Magnetic Symmetry

- All physical quantities must transform appropriately under these symmetries
- **Example:** Net magnetization should be *invariant*

$$\boldsymbol{M} \xrightarrow{C_{4z}+T} -M_y\hat{\boldsymbol{x}} + M_x\hat{\boldsymbol{y}} + M_z\hat{\boldsymbol{z}} \xrightarrow{T} M_y\hat{\boldsymbol{x}} - M_x\hat{\boldsymbol{y}} - M_z\hat{\boldsymbol{z}}$$

- Implies immediately that $\boldsymbol{M} = \boldsymbol{0}$.
- Symmetry *requires* that MnF_2 is compensated; similarly **AHE vanishes**
- Also implies that $N_x = N_y = 0$ – Neel vector must be along ***z***

Piezomagnetism

- Other observables can be analyzed in same way; *piezomagnetism* is magnetization induced by applied strain

$$\boldsymbol{M} = \sum_{\mu\nu} \boldsymbol{C}_{\mu\nu} \epsilon_{\mu\nu}$$

- Strain transforms like a tensor $\boldsymbol{\epsilon} \rightarrow \boldsymbol{R}^{-1} \boldsymbol{\epsilon} \boldsymbol{R}$

$$\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}, \epsilon_{yz}, \epsilon_{xz}, \epsilon_{xy} \xrightarrow{C_{4z}+T} \epsilon_{yy}, \epsilon_{xx}, \epsilon_{zz}, -\epsilon_{xz}, \epsilon_{yz}, -\epsilon_{xy}$$

- Using all these symmetries: $\boldsymbol{M} = C' \left(\epsilon_{yz} \hat{\boldsymbol{x}} + \epsilon_{xz} \hat{\boldsymbol{y}} \right) + C \epsilon_{xy} \hat{\boldsymbol{z}}$

Dzialoshinskii, JETP **33**, 807 (1958)

Other examples are the antiferromagnetics MnF_2, CoF_2, and FeF_2. In accordance with Ref. 4, their magnetic symmetry class consists of

$$C_2,\ 2C_4R,\ 2U_2,\ 2U_2'R,\ I,\ \sigma_h,\ 2S_4R,\ 2\sigma_v,\ 2\sigma_v'R.$$

This symmetry group leaves invariant the following term in the expression for Φ:

$$\Phi = -\lambda(\sigma_{xz}H_y + \sigma_{yz}H_x),$$

whence we get for the magnetic moment

$$m_x = \lambda\sigma_{yz},\quad m_y = \lambda\sigma_{xz}.$$

THE PROBLEM OF PIEZOMAGNETISM

I. E. DZIALOSHINSKII

Physical Problems Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor June 20, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 807-808 (September, 1957)

IN the recent literature one encounters assertions that piezomagnetic bodies in general cannot exist in nature. This conclusion has been based on the invariance of the equations of mechanics with respect to

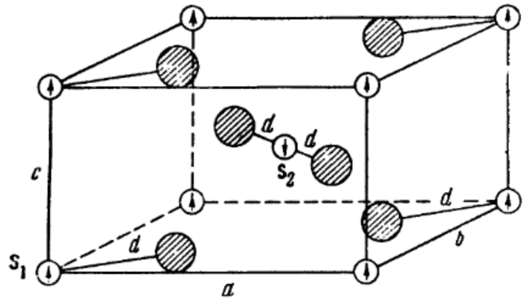


FIG. 1. The magnetic structure of MnF_2 and CoF_2. Open circles – Mn, Co; shaded circles – F.

*In Dzyaloshinskiĭ paper[5] a term was omitted in the expression for the thermodynamic potential responsible for the appearance of a piezo-magnetic moment along the z axis. A complete analysis of this problem was given in his dissertation.[18]

but only on applying shear stresses

$$m_x^p = \Lambda_1 \sigma_{yz}, \qquad m_y^p = \Lambda_1 \sigma_{xz}, \qquad m_z^p = \Lambda_2 \sigma_{xy}. \qquad (1)$$

SOVIET PHYSICS JETP VOLUME 11, NUMBER 4 OCTOBER, 1960

PIEZOMAGNETISM IN THE ANTIFERROMAGNETIC FLUORIDES OF COBALT AND MANGANESE

A. S. BOROVIK-ROMANOV

Institute of Physical Problems, Academy of Sciences, U.S.S.R.

Submitted to JETP editor November 6, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 1088-1098 (April, 1960)

A special magnetic balance and press were constructed to observe piezomagnetism experimentally. In agreement with theoretical predictions, piezomagnetic moments m_z^p were found

Multipoles

- Only symmetry distinct if *spherically symmetric*
- A definition of a magnetic octupole

Always induce higher multipoles in crystal

$$\boldsymbol{O}_{\mu\nu} \equiv \int d^3r\; r_\mu r_\nu \boldsymbol{m}(\boldsymbol{r})$$

$$O^z_{xy} \xrightarrow{C_{4z}+T} +O^z_{xy}$$

$$O^z_{zz} \xrightarrow{C_{4z}+T} -O^z_{zz}$$

$$O^x_{yz} \xrightarrow{C_{4z}+T} -O^y_{xz}$$

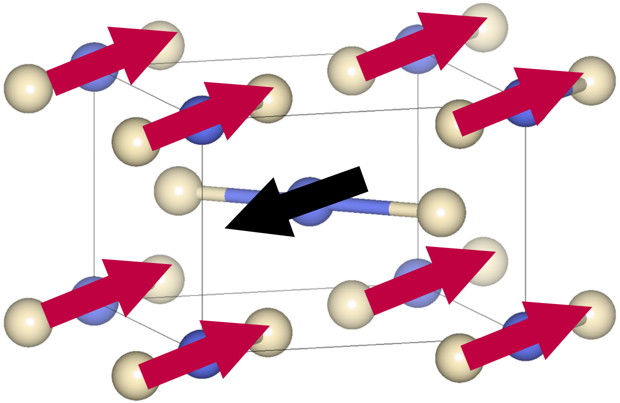
- Two fully symmetric components can be constructed after apply all operations

$$O^z_{xy}, \quad O^x_{yz} + O^y_{xz}$$

Expect to be non-zero in the ordered phase

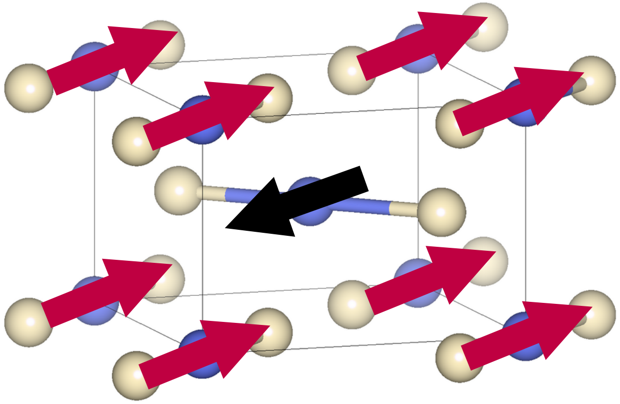
Bhowal & Spaldin, Phys. Rev. X **14**, 011019 (2024)

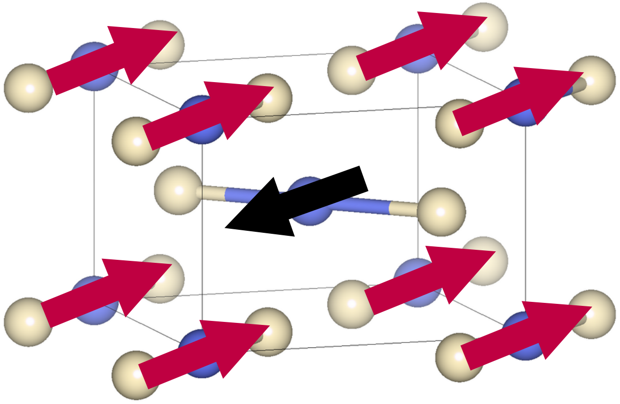
Symmetry depend on **moment direction**



$C_{2,x+y}$

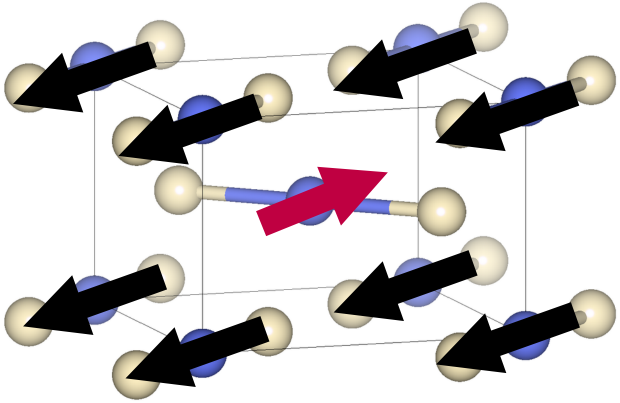
… still a symmetry





C_{2z}

… with T.R.



… rest of four-fold and two-fold operations are simply *not symmetries*

Magnetic Symmetry (cont.)

- Moment *direction* affects the magnetic space group
- Net magnetization is less constrained by symmetry for in-plane $\boldsymbol{N}$

$$\boldsymbol{M} \xrightarrow{C_{2z}+T} M_x\hat{\boldsymbol{x}} + M_y\hat{\boldsymbol{y}} - M_z\hat{\boldsymbol{z}} \qquad M_z = 0$$

$$\boldsymbol{M} \xrightarrow{C_{2,x-y}+T} M_y\hat{\boldsymbol{x}} + M_x\hat{\boldsymbol{y}} \qquad M_x = M_y \equiv M$$

$$\boldsymbol{M} \xrightarrow{C_{2,x+y}} \frac{M}{\sqrt{2}}(\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}})$$

(weak) Ferromagnet

not antiferromagnetic

Simpler antiferromagnet

- Consider a case where these *are* identical: we have a **translation** connecting the two sublattices

$$\boldsymbol{M} \xrightarrow{\tau+T} -\boldsymbol{M} \quad \Rightarrow \quad \boldsymbol{M} = 0$$

$$\boldsymbol{M} = \sum_{\mu\nu} \boldsymbol{C}_{\mu\nu}\epsilon_{\mu\nu} \xrightarrow{\tau+T} -\boldsymbol{M} = \sum_{\mu\nu} \boldsymbol{C}_{\mu\nu}\epsilon_{\mu\nu} \quad \Rightarrow \quad \boldsymbol{C}_{\mu\nu} = 0$$

$$\boldsymbol{O}_{\mu\nu} \xrightarrow{\tau+T} -\boldsymbol{O}_{\mu\nu} \quad \Rightarrow \quad \boldsymbol{O}_{\mu\nu} = 0$$

Interlude

- So for a given crystal and magnetic structure, can work out a **magnetic space group**
- There are **1651 distinct magnetic space groups** (well understood)
  - Many more that the usual 230 space groups
- So are altermagnets just some subset of these groups?
  - Some anti-ferromagnetic (zero moment) some weak ferromagnets?
  - Was this just hidden in MSG tables from the 1960s and no one noticed?
  - What is *new* in this definition of an altermagnet?

Answer: Yes *and* no

Are altermagnets *really new?*

- **Altermagnets are well-defined in the non-relativistic limit** (*zero* spin-orbit coupling, *zero* dipolar interactions)
  - In this limit they are distinct from FMs and AFMs
  - Not a ferromagnet since zero moment
  - Symmetry acts differently than simplest AFMs
- Symmetries are higher in non-relativistic limit: **Spin space groups**
  - Spin rotations can act independently than spatial symmetries
  - Classification of goes beyond usual 230 space groups or the 1651 magnetic space groups

Brinkman & Elliot, Proc. Roy. Soc. A **294**, 343 (1966), Litvin & Opechowski, Physica **76,** 538 (1974), Corticelli *et al,* Phys. Rev. B 105, 064430 (2022), Xiao *et al* Phys. Rev. X 14, 031037 (2024), Chen *et al,* Phys. Rev. X 14, 031038 (2024), ...

Questions

- Is this really a **third kind of magnetism?** No
  - *Why only two? What about spirals and non-colinear or coplanar orders, etc*
- Is this more like a ferromagnet or more like an antiferromagnet? Yes (no SOC) No (w/ SOC)
  - *When is it really distinct from either of those?*
- How does this relate to large body of knowledge on magnetic symmetries?
  - *Like space groups, there are magnetic space groups that have been well-understood for decades*

Spin Space Groups (no SOC) *With SOC? Magnetic groups*

Non-relativistic symmetries

- Altermagnets *are* distinct in the non-relativistic limit
  - Absent spin-orbit coupling, dipole-dipole interactions, crystal field effects
- Lots of magnetic materials *naturally* near this limit
  - Elements high up the periodic table (Cu, Ni, …)
  - Elements with spin only moments (Mn^{2+}, Fe^{3+}, Eu^{2+}, Gd^{3+}, …)
  - … sometimes it happens accidentally (e.g. some Yb^{3+} compounds)
- **Never actually zero**
  - *Essential* to delineate relativistic and non-relativistic contributions

Non-relativistic symmetries

- Without spin orbit coupling, **spin and space can transform independently**

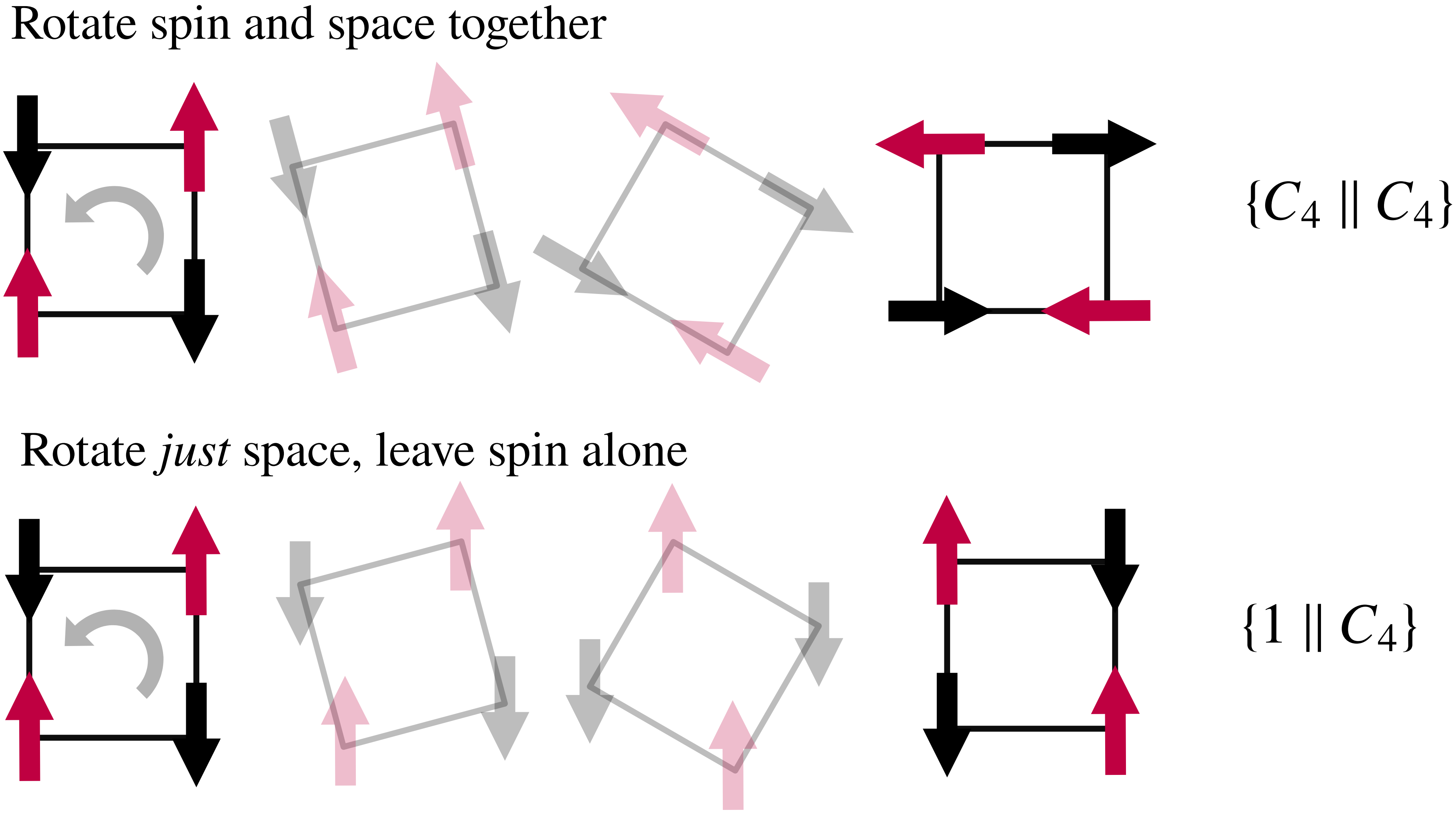
$$\{\pm \boldsymbol{R}, \boldsymbol{\tau}\}[\boldsymbol{m}(\boldsymbol{r})] = \boldsymbol{R}\boldsymbol{m}(\pm \boldsymbol{R}^{-1}(\boldsymbol{r} - \boldsymbol{\tau}))$$

Rotation & translation in space

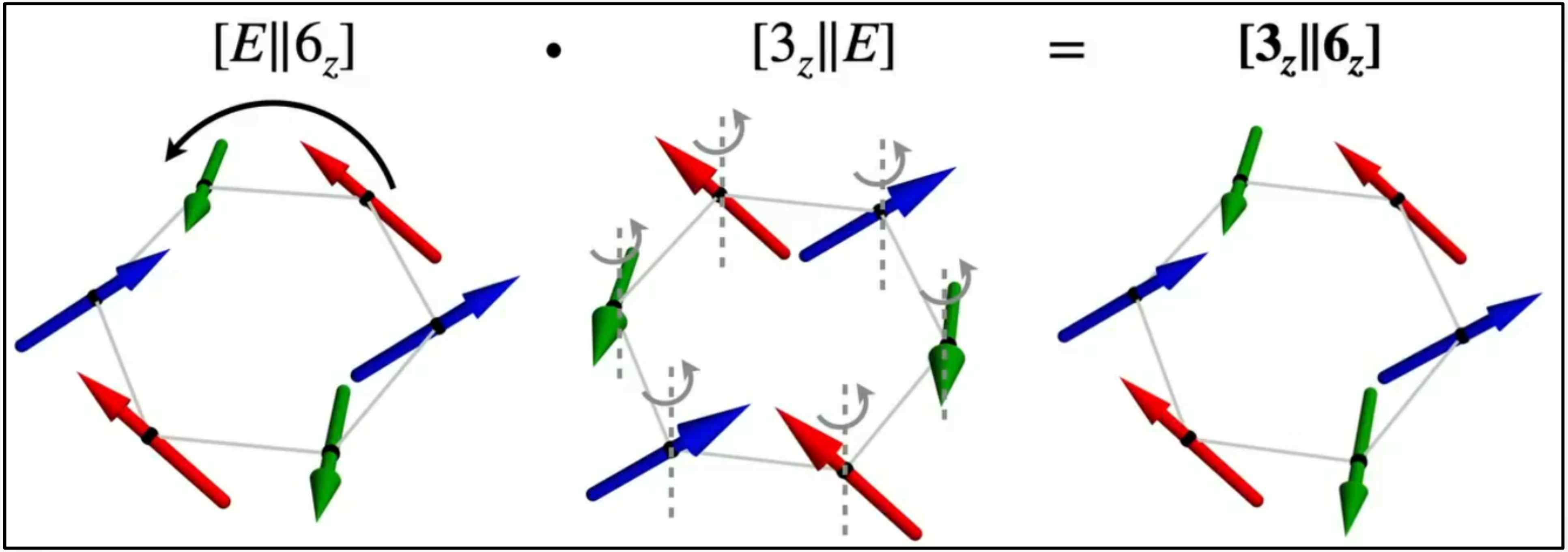
$$\{\boldsymbol{M} || \pm \boldsymbol{R}, \boldsymbol{\tau}\}[\boldsymbol{m}(\boldsymbol{r})] = \boldsymbol{M}\boldsymbol{m}(\pm \boldsymbol{R}^{-1}(\boldsymbol{r} - \boldsymbol{\tau}))$$

Separate rotation on spin

- Symmetry groups of this form are called **Spin Space Groups** (for infinite lattices) and **Spin Point Groups** (for finite objects)


Rotate spin and space together
$\{C_4 \parallel C_4\}$
Rotate *just* space, leave spin alone
$\{1 \parallel C_4\}$

Example* ${}^{3_z}6/{}^{1}m^{m_x}m^{m_{xy}}m$



* stolen from H. Schiff

Atoms related by 60 degree rotation,
spins by 120 degree rotation

Classification of Spin Space Groups

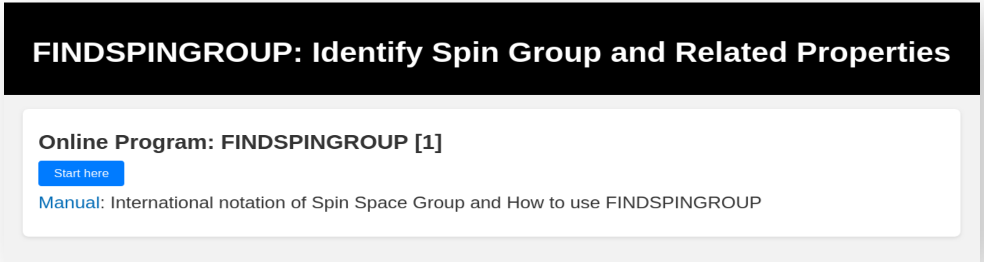
- *Infinite* number of spin space groups; Partial classifications from several groups
  - **Colinear orders:** 1421[a,b,c]
  - **Coplanar orders:** 16383[a] *or* 9542[b] *or* 24788[c]
  - **Non-coplanar orders:** 87308[a] *or* 56512[b] *or* 157289[c]

Q. Liu's group[a]
(Supercells size 8)
Z. Song's group[b]
(Families mod ***k***)
C. Fang's group[c]
(Supercells size 12)

- Colinear case is simplest

Xiao *et al* Phys. Rev. X 14, 031037 (2024), Chen *et al*, Phys. Rev. X 14, 031038 (2024), Jiang *et al*, Phys. Rev. X (2024), …

https://findspingroup.com/



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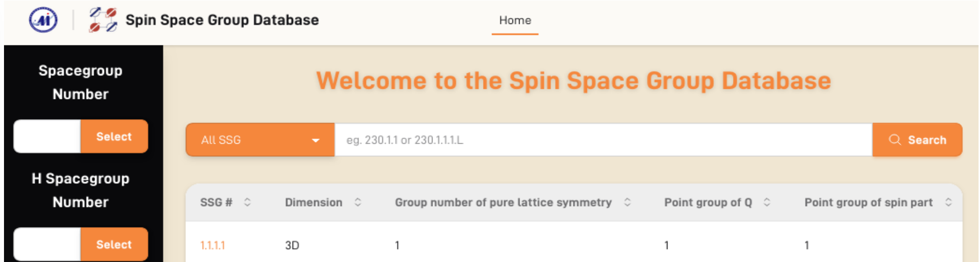
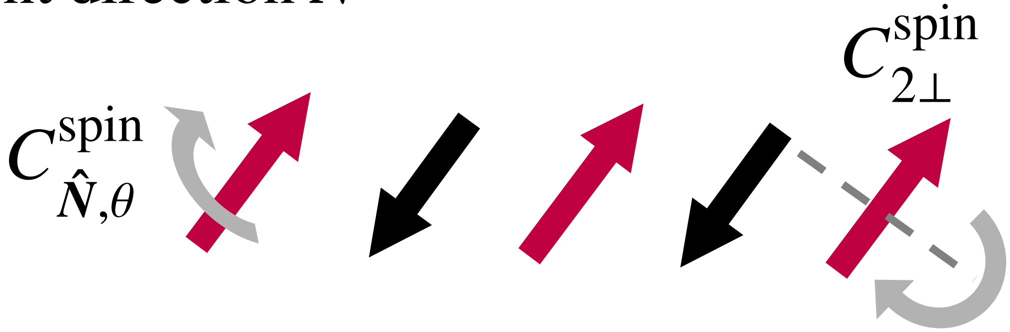


Colinear Spin Space Groups

- Altermagnets are *colinear*; subset of these 1421 SSGs
- All colinear spin space groups have $U(1)$ continuous symmetry along the moment direction N



$$\boldsymbol{m}_i = \pm \boldsymbol{N}$$

- Projection of spin along this axis is good quantum number
- Additional symmetry in all these groups $C^{\text{spin}}_{2\perp} + \text{T.R.}$

Types of colinear SSGs

Šmejkal *et al* Phys. Rev. X **12**, 031042 (2022), Šmejkal *et al*, Phys. Rev. X **12**, 040501 (2022), Xiao *et al* Phys. Rev. X 14, 031037 (2024), Chen *et al*, Phys. Rev. X 14, 031038 (2024), Jiang *et al*, Phys. Rev. X (2024), ...

- These can be grouped into three types:

$\{1 \parallel g\}, g \in \mathrm{SG}$ 230 groups **Ferromagnets**

$\{1 \parallel g\}$ or $\{C_{2,\perp} \parallel gh\},\ g \in \mathrm{SG} \bmod h$ *Zero dipole moment*

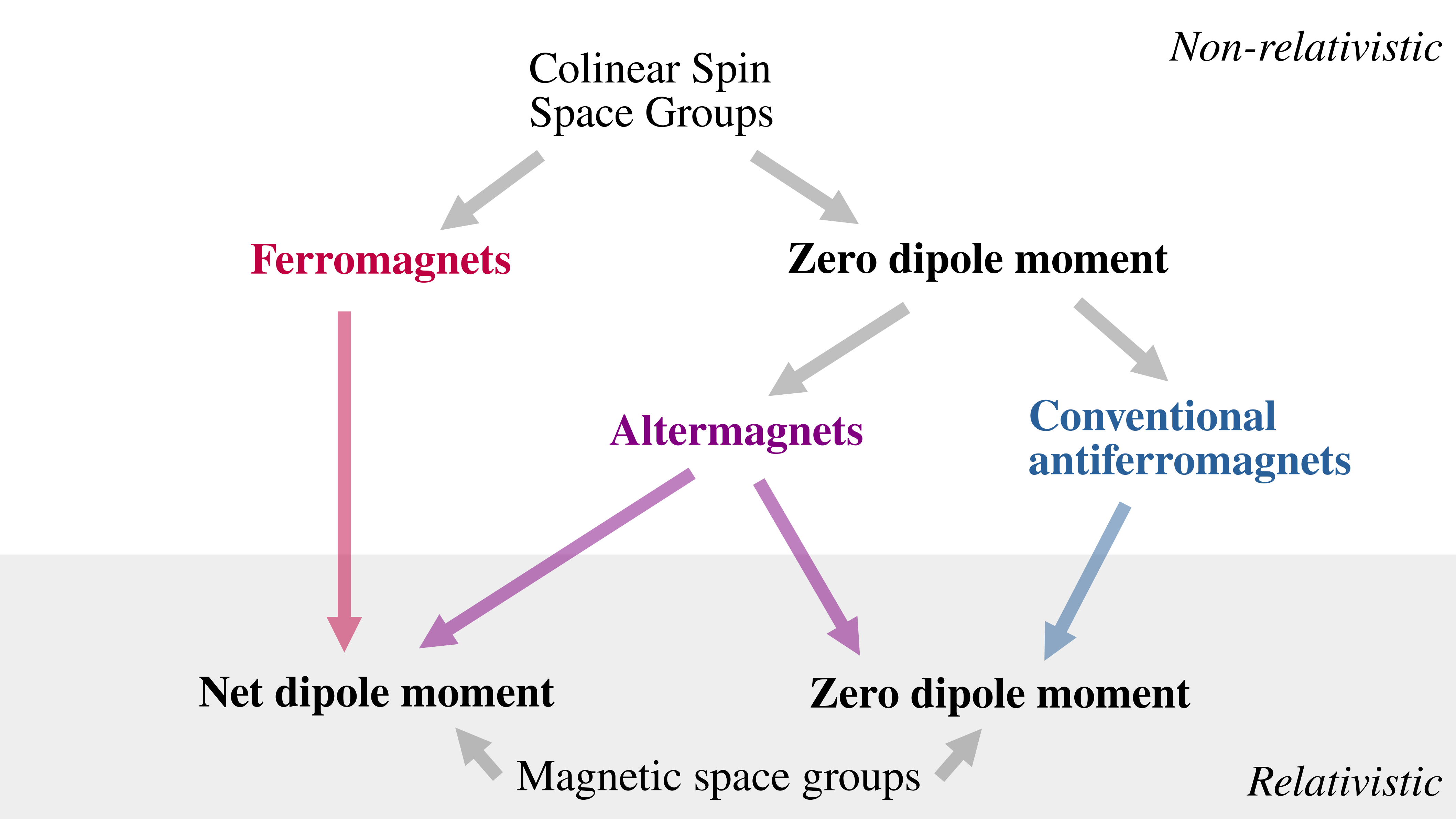
Rotation or reflection

674 groups

Altermagnets

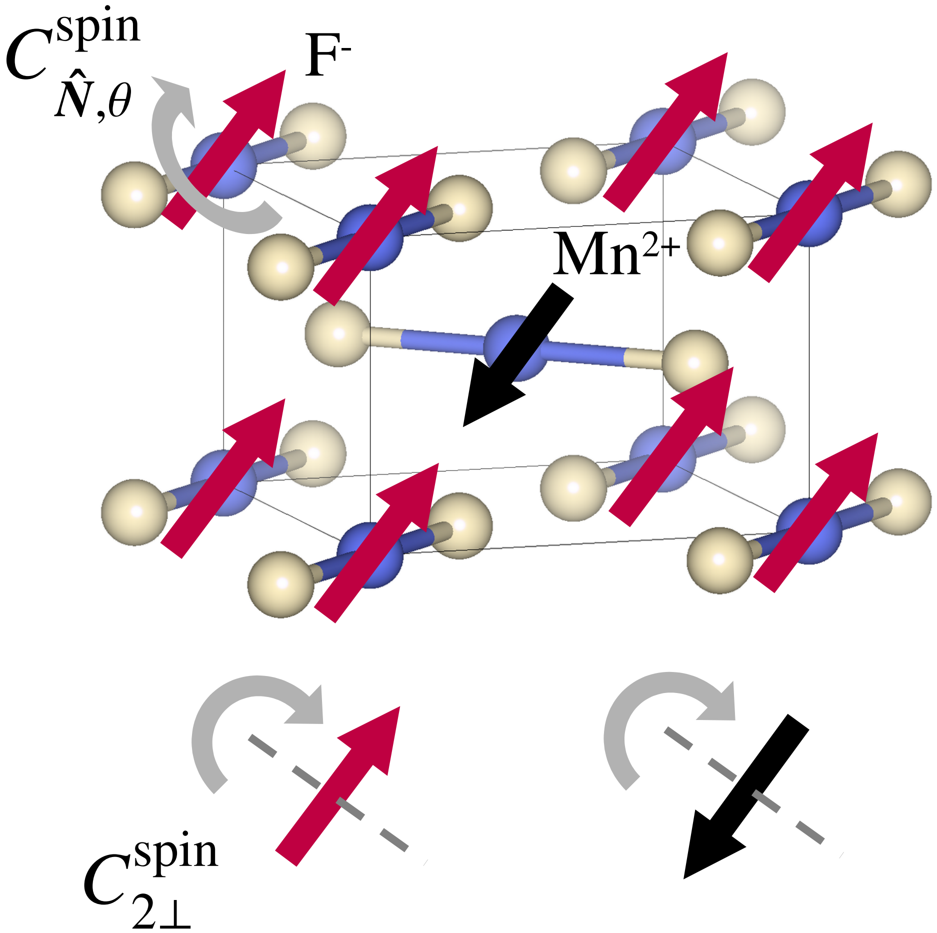
Translation or inversion

517 groups **Conventional Antiferromagnets**


Non-relativistic
Colinear Spin Space Groups
Ferromagnets
Zero dipole moment
Altermagnets
Conventional antiferromagnets
Net dipole moment
Zero dipole moment
Magnetic space groups
Relativistic

Example: *MnF*$_2$

- Spatial operations don't change the spins anymore!
- Two possibilities:
  - Swaps sublattices: $C_{4z}+\tau$, $C_{2,x+y}$
  - Doesn't swap: $C_{2x}+\tau$, C_{2z}
- Follow those that swap with **spin rotation** $\perp N$ *or* time reversal
- Still have continuous spin rotations about N and two-fold spin rotation + T. R.



Example: MnF_2

$$\{C_{2,\perp} \parallel C_{4z} + \tau\} \ \{C_{2,\perp} \parallel C_{2,x+y}\} \ \{1 \parallel C_{2x} + \tau\} \ \{1 \parallel C_{2z}\}$$

- Symmetries are *much more constraining*

$$\boldsymbol{M} \xrightarrow{\{C_{\hat{\mathbf{N}},\theta} \parallel 1\}} \boldsymbol{R}_{\hat{N}}(\theta)\boldsymbol{M} \qquad \boldsymbol{M} = M\hat{\mathbf{N}}$$

$$\boldsymbol{M} \xrightarrow{\{C_{2,\perp} \parallel C_{4z}+\tau\}} \boldsymbol{M} \xrightarrow{\{1 \parallel C_{2\perp}\}} -M\hat{\mathbf{N}} \qquad \boldsymbol{M} = \mathbf{0}$$

- *Always* compensated in non-relativistic limit, independent of direction

Multipoles

$$\boldsymbol{O}_{\mu\nu} \equiv \int d^3r\; r_\mu r_\nu \boldsymbol{m}(\boldsymbol{r})$$

- Do *not* rule out induced multipoles

$$\boldsymbol{O}_{\mu\nu} \xrightarrow{\{C_{\hat{\mathbf{N}},\theta} \,\|\, 1\}} \boldsymbol{R}_{\hat{\mathbf{N}}}(\theta)\boldsymbol{O}_{\mu\nu} \quad \Rightarrow \quad \boldsymbol{O}_{\mu\nu} = O_{\mu\nu}\hat{\mathbf{N}}$$

$$O_{xx}, O_{yy}, O_{yz}, O_{xz}, O_{xy}$$

$$\xrightarrow{\{C_{2,\perp} \,\|\, C_{4z}+\tau\}}$$

$$-O_{yy}, -O_{xx}, -O_{xz}, +O_{yz}, +O_{xy}$$

$$\Rightarrow \quad O_{xx} = -O_{yy}$$

$$\Rightarrow \quad O_{yz} = O_{xz} = 0$$

- Remaining symmetries eliminate O_{xx}

$$\boldsymbol{O}_{xy} \propto \hat{\mathbf{N}}$$

Piezomagnetism

$$\boldsymbol{M} = \sum_{\mu\nu} \boldsymbol{C}_{\mu\nu}\epsilon_{\mu\nu}$$

- Does *not* rule out piezomagnetism

$$\boldsymbol{M} \xrightarrow{\{C_{\hat{\boldsymbol{N}},\theta} \,\|\, 1\}} \boldsymbol{R}_{\hat{\boldsymbol{N}}}(\theta)\boldsymbol{M} \Rightarrow \boldsymbol{M} = M\hat{\boldsymbol{N}}$$

$$\epsilon_{\mu\nu} \xrightarrow{\{C_{\hat{\boldsymbol{N}},\theta} \,\|\, 1\}} \epsilon_{\mu\nu} \Rightarrow \boldsymbol{C}_{\mu\nu} = C_{\mu\nu}\hat{\boldsymbol{N}}$$

- Strain transforms almost identically to $\boldsymbol{O}$

$$\boldsymbol{M} = C\epsilon_{xy}\hat{\boldsymbol{N}}$$

$$\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}, \epsilon_{yz}, \epsilon_{xz}, \epsilon_{xy} \xrightarrow{\{C_{2,\perp} \,\|\, C_{4z}+\tau\}} \epsilon_{yy}, \epsilon_{xx}, \epsilon_{zz}, -\epsilon_{xz}, \epsilon_{yz}, -\epsilon_{xy}$$

Implications

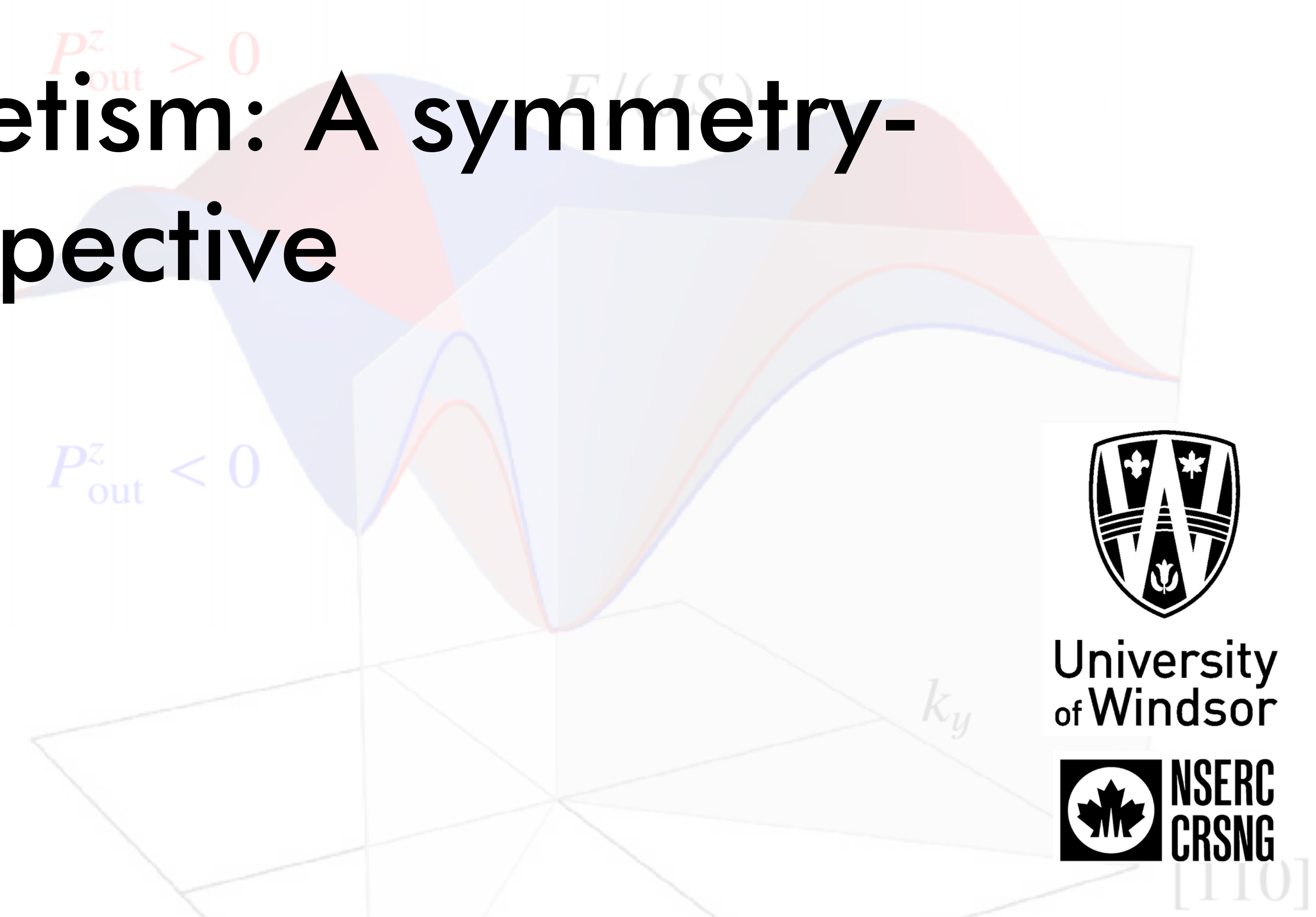
- Spin symmetries remove *some* of these responses, but all of them
  - Piezomagnetism remains
  - Magnetic octupoles remain
  - Spin-splitting remains
- Can be understood using spin space groups & spin point groups
- **Prescription is clear:**
  - Classify observables, states, bands, … in terms representations of spin groups
  - Make predictions in the spin-orbit free limit

Take-aways

- Altermagnets *are* a new class of magnetic ordering, **in the non-relativistic limit**
  - *Not* once relativistic effects are included, *but many properties of non-relativistic limit are dictated by non-relativistic case*
- Properties of altermagnets are (mostly) determined by symmetries of the non-relativistic limit: **spin groups**
  - Symmetries where operations on space and on spin act *independently*
- *Some* of the properties of altermagnets can be understood from the paramagnetic phase, avoiding spin groups

Tutorial:

Altermagnetism: A symmetry-based perspective



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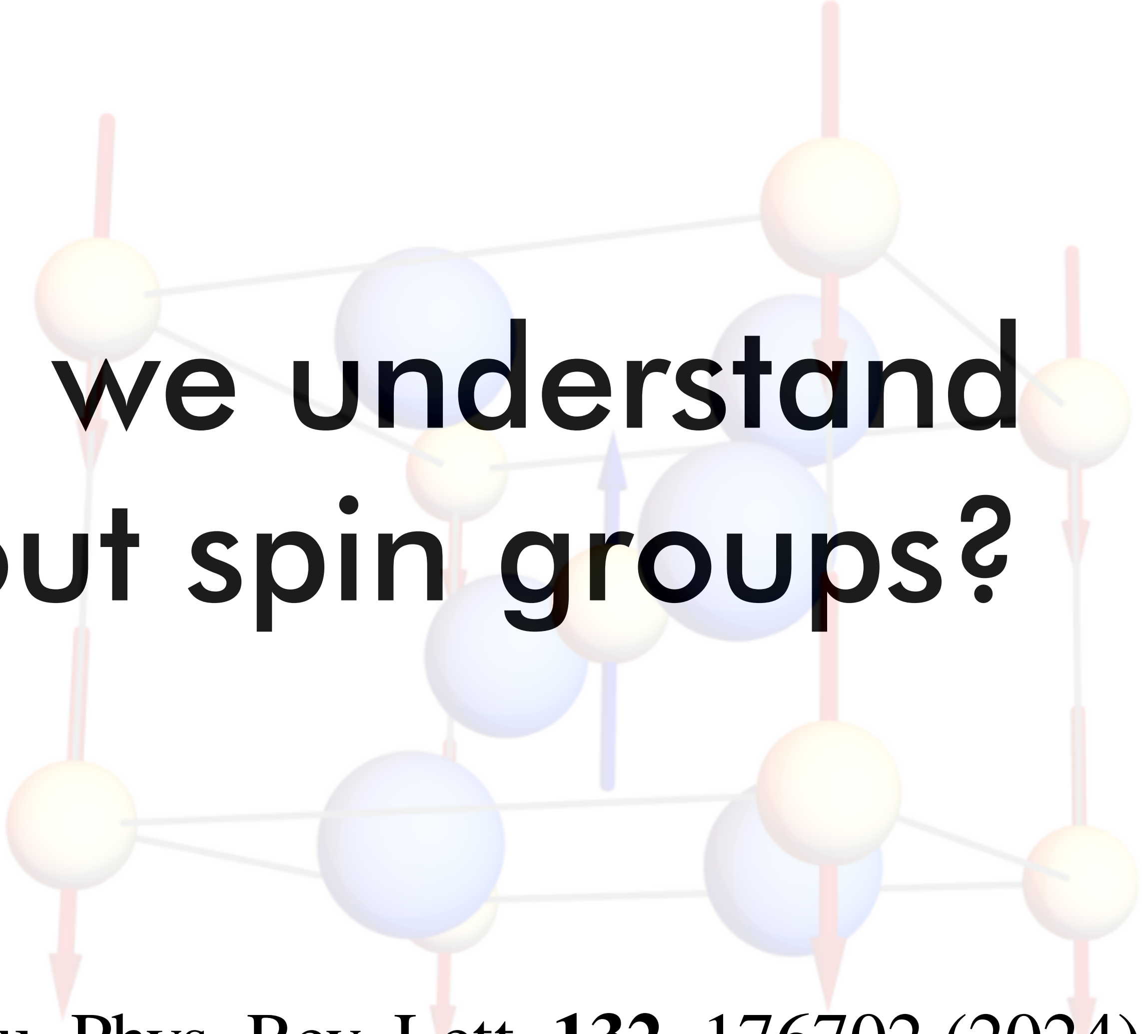
NSERC CRSNG

Outline

1. *Landau theory*
2. Insulating altermagnets
3. Altermagnetic chiral magnons
4. Detection, realization & characterization



What can we understand this without spin groups?

P. A. McClarty & J. G. Rau, Phys. Rev. Lett. **132**, 176702 (2024) ❦

Definition via Landau Theory

Don't need much of the more detailed formalism

- Start from the **paramagnetic** phase work at $\boldsymbol{k} = 0$
  - Symmetries are point group symmetries & $SO(3)$ spin rotations

Definition: Néel vector transforms **as non-trivial one-dimensional irrep** (inversion even) under spatial symmetries

Néel Vector →

$$\boldsymbol{N} \sim \Gamma_N \otimes \Gamma_A^S$$

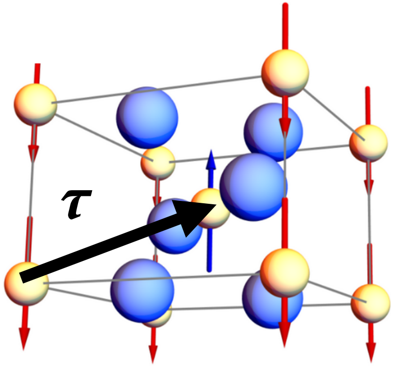
← Vector under spin-rotation symmetry

1D irrep under spatial symmetry

Note: Assume $\boldsymbol{k} = 0$, and so no translation symmetries that connect the sublattices.

Example: *Rutiles*

- Space group is $P4_2/mnm$ (#136) with point group D_{4h}
- Magnetic ion at Wyckoff position *2a*
  - No (pure) translation connecting *A* & *B* sublattice, *2a* sites are inversion centers
- Identify irrep of (spatial) part of Néel order from standard tables
- Néel vector transforms as $N \sim B_{2g}$ (spatially)



These two operations have translations in SG

D4h	E	2C4 (z) +τ	C2	2C'2 +τ	2C''2
A_{1g}	+1	+1	+1	+1	+1
A_{2g}	+1	+1	+1	-1	-1
B_{1g}	+1	-1	+1	+1	-1
B_{2g}	+1	-1	+1	-1	+1
E_g	+2	0	-2	0	0

Landau Free Energy

- Landau theory is *trivial* without spin-orbit coupling

$$\Phi = a_2 \boldsymbol{N}\cdot\boldsymbol{N} + a_4(\boldsymbol{N}\cdot\boldsymbol{N})^2 + \ldots$$

Forced by spin-rotation symmetry

- … but can ask questions about couplings to *other* observables (say, $\boldsymbol{X}$)

Irrep of $\boldsymbol{X}$

$$\boldsymbol{X}\cdot\boldsymbol{N} \sim (\Gamma_X \otimes \Gamma_A^S) \otimes (\Gamma_N \otimes \Gamma_A^S)$$

$$= \Gamma_1 \otimes \Gamma_1^S + \ldots$$

Irrep of $\boldsymbol{N}$

If contains trivial irrep ...

… linear term is allowed

$$F(\boldsymbol{N},\boldsymbol{X}) = -b\boldsymbol{X}\cdot\boldsymbol{N} + \frac{a}{2}|\boldsymbol{X}|^2$$

Minimization of F gives $\boldsymbol{X}$ proportional to $\boldsymbol{N}$

$$\boldsymbol{X} = \frac{b}{a}\boldsymbol{N}$$

$$\boldsymbol{X} \sim \Gamma_X \otimes \Gamma_A^S$$

Magnetization

- Consider the magnetization vector $\boldsymbol{M}$
- Transforms *trivially* under spatial symmetries, *vector* in spin space
- Can this couple *linearly* to the Neel vector?

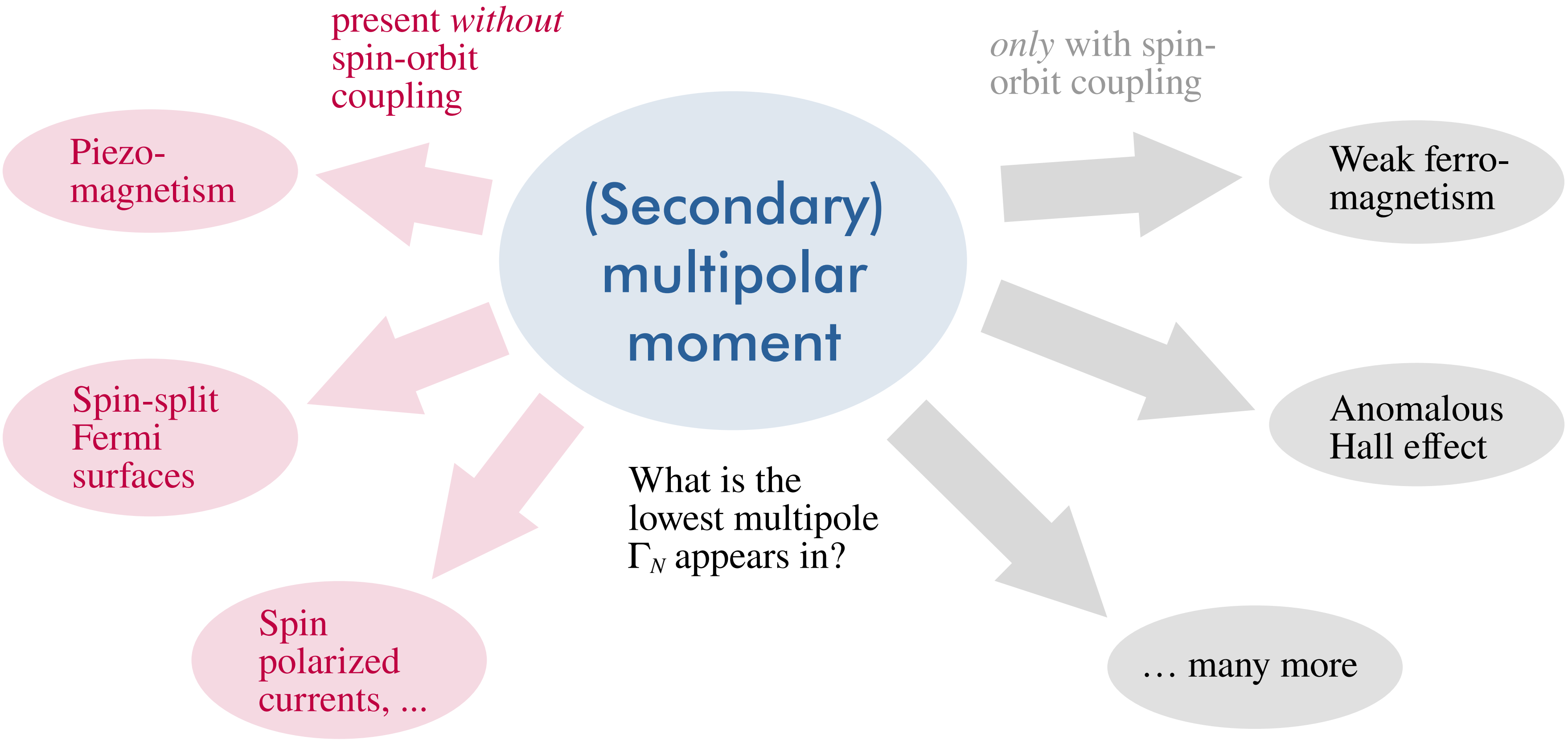
Dot product invariant under spin rotations

$$(\Gamma_N \otimes \Gamma_A^S) \otimes (\Gamma_1 \otimes \Gamma_A^S) = (\Gamma_N \otimes \Gamma_1) \otimes (\Gamma_1^S \oplus \Gamma_A^S \oplus \Gamma_Q^S)$$

Irrep of $\boldsymbol{N}$ — Irrep of $\boldsymbol{M}$ — **Spatial part cannot contain trivial irrep**

- No ferro- or ferri-magnetic moment *necessarily* induced

Symmetry forbids linear coupling to magnetization vector



Presence of *all* these responses dictated by multipolar character of the irrep Γ_N

Secondary Order Parameters

- Consider the magnetic *octupole* $\boldsymbol{O}_{\mu\nu} = \int d^3r\, r_\mu r_\nu \boldsymbol{m}(\boldsymbol{r})$
- Transforms as spatial *quadrupole* times spin-vector $\boldsymbol{O}_{\mu\nu} \sim \Gamma_Q \otimes \Gamma_A^S$
- Can this couple *linearly* to the Neel vector?

$$(\Gamma_N \otimes \Gamma_A^S) \otimes (\Gamma_Q \otimes \Gamma_A^S) = (\Gamma_N \otimes \Gamma_Q) \otimes (\Gamma_1^S \oplus \Gamma_A^S \oplus \Gamma_Q^S)$$

Irrep of $\boldsymbol{N}$ — Irrep of $\boldsymbol{O}$ — Contains trivial irrep?

- If so, free energy must contain term: $\propto \boldsymbol{N} \cdot \boldsymbol{O}_{\mu\nu}$

Always induces finite octupole moment

Secondary octupolar order parameter is present if Γ_Q contains Γ_N

* Bhowal & Spaldin, Phys. Rev. X 14, 011019 (2024)

Piezo-magnetism

- Presence of magnetic octupole dictates other observables

Elastic strain

$$\epsilon_{\mu\nu} = \Gamma_Q \otimes \Gamma_1^S$$

Trivial under spin-rotation

Spatial quadrupole

$$H \sim \Gamma_1 \otimes \Gamma_A^S$$

(Same as ***M***)

- *Piezo-magnetism* is (trilinear) coupling between ***N***, ***H*** and ε
  - … existence tied to *same* condition as magnetic octupole

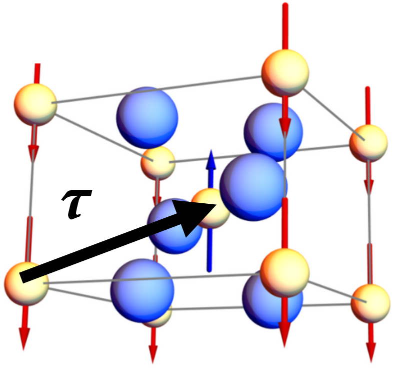
Secondary octupolar order parameter (Γ_Q contains Γ_N) implies the presence of piezo-magnetism

Responses without spin-rotation symmetry?

Dzialoshinskii, Sov. Phys. JETP **6,** 621-622 (1958)

Application: *Rutiles*

- Space group is $P4_2/mmm$ (#136) with point group D_{4h}
- Magnetic ion at Wyckoff position *2a*
  - No (pure) translation connecting *A* & *B* sublattice, *2a* sites are inversion centers
- Identify irrep of (spatial) part of Néel order from standard tables
- Néel vector transforms as $N \sim B_{2g}$ (spatially)



These two operations have translations in SG

D_{4h}	E	2C$_4$ (z) $+\tau$	C$_2$	2C'$_2$ $+\tau$	2C''$_2$
A_{1g}	+1	+1	+1	+1	+1
A_{2g}	+1	+1	+1	-1	-1
B_{1g}	+1	-1	+1	+1	-1
B_{2g}	+1	-1	+1	-1	+1
E_g	+2	0	-2	0	0

Application: MX_2

- *Spatial* quadrupole contains Néel irrep? **Yes**

$$\Gamma_Q = \overset{z^2}{A_{1g}} \oplus \overset{x^2\text{-}y^2}{B_{1g}} \oplus \underbrace{\overset{xy}{B_{2g}}}_{\Gamma_N} \oplus \overset{xz,\ yz}{E_g}$$

- Thus Néel vector linearly couples to quadrupole: **secondary order parameter**

$$\propto \boldsymbol{N} \cdot \boldsymbol{O}_{xy}$$

- Piezomagnetism in absence of spin-orbit coupling implied immediately

What about the spin-splitting?

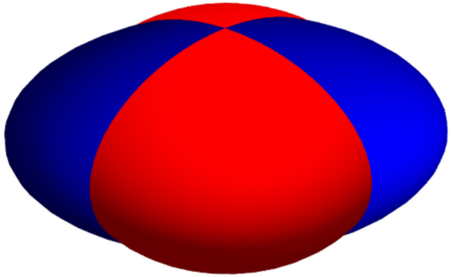
Application: MX_2

Same symmetry
$k_x k_y \boldsymbol{\sigma} \leftrightarrow r_x r_y \boldsymbol{m}(\boldsymbol{r})$

- Consider the long-wavelength limit near $\boldsymbol{k} = 0$
- What can $\boldsymbol{N}$ couple to linearly in Hamiltonian?*

$$H(\boldsymbol{k}) = \epsilon_0(\boldsymbol{k}) + \alpha k_x k_y \boldsymbol{\sigma} \cdot \boldsymbol{N} + \cdots$$

- Symmetries of band Hamiltonian mirror those of the magnetic octupole!
  - Substitute: $\boldsymbol{k} \Leftrightarrow \boldsymbol{r}$ and $\boldsymbol{m}(\boldsymbol{r}) \Leftrightarrow \boldsymbol{\sigma}$



***d*-wave spin-splitting** mirrors the leading coupling to **quadrupole**

*Project into isolated pair of bands (spin up and spin down)

Lesson: "Secondary" order parameter sets the spin-splitting pattern

With spin-orbit coupling?

Anomalous Hall and Weak FM

- Adding spin-orbit means promoting spin irreps to spatial irreps
- Can make *general* statements about linear couplings to ***M***, ***H***, σ_H

$$(\Gamma_1 \otimes \Gamma_A) \otimes (\Gamma_N \otimes \Gamma_A) = \Gamma_N \otimes (\Gamma_A \otimes \Gamma_A)$$

Irrep of ***M, H*** or σ_H — Irrep of ***N*** — Contains trivial irrep?

- Since $\Gamma_A \otimes \Gamma_A = \Gamma_1 \oplus \Gamma_A \oplus \Gamma_Q$ true *if* Γ_Q or Γ_A contains Γ_N

Secondary octupolar order parameter (Γ_Q contains Γ_N) implies weak ferromagnetism **and*** an anomalous Hall effect (once spin-orbit coupling is included)

*Dependent on direction of *N*

Application: MX_2

- The spin vectors transform as *axial* vectors

$$\Gamma_A^S \to A_{2g} \oplus E_g$$

- Returning to "usual" symmetry analysis we have:

$$\boldsymbol{N} \sim B_{2g} \otimes (A_{2g} \oplus E_g) \sim B_{1g} \oplus E_g$$

B_{1g}: N_z; E_g: N_x, N_y

$$\boldsymbol{M} \sim A_{1g} \otimes (A_{2g} \oplus E_g) \sim A_{2g} \oplus E_g$$

A_{2g}: M_z; E_g: M_x, M_y

AHE vector transforms identically to $\boldsymbol{M}$

- Weak ferromagnetism? **Yes***
- Anomalous Hall? **Yes*** σ_{yz} and σ_{xz} components

*For in-plane Néel vector

Application: *Hexagonal MnTe*

- Néel vector transforms as $N \sim B_{1g}$
- *Spatial* quadrupole contains Néel? **No**

$$\Gamma_N \not\subset \Gamma_Q$$

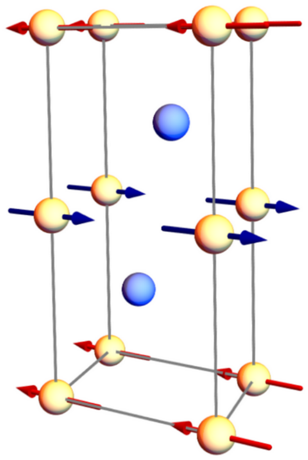
Higher multipole **does**

- Secondary octupolar order parameter? **No!**
- Look at higher multipole containing Γ_N
- **Not a quadrupole, but hexadecapole!**

$$\propto N \cdot O_3^4$$

Secondary order parameter

$$O_3^4 \equiv \int d^3r\,(Y_3^4(\hat{r}) - Y_{-3}^4(\hat{r}))m(r)$$



Point group is D_{6h}

+τ +τ +τ

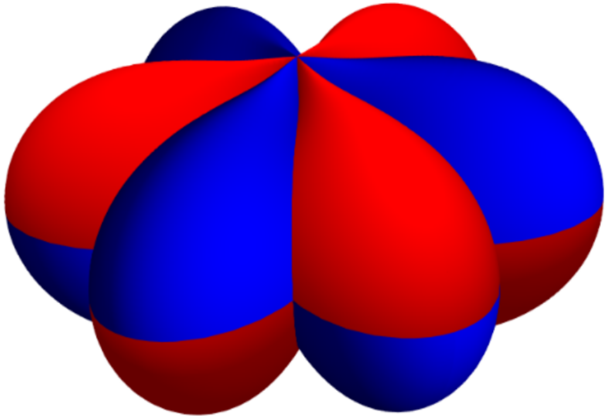
D_{6h}	E	$2C_6$ (z)	$2C_3$	C_2	$3C'_2$	$3C''_2$
A_{1g}	+1	+1	+1	+1	+1	+1
A_{2g}	+1	+1	+1	+1	-1	-1
B_{1g}	+1	-1	+1	-1	+1	-1
B_{2g}	+1	-1	+1	-1	-1	+1
E_{1g}	+2	+1	-1	-2	0	0
E_{2g}	+2	-1	-1	+2	0	0

Application: *Hexagonal MnTe*

- *Higher* multipole determines spin-splitting of Fermi surface
- In this case: ***g*-wave splitting (*l* = 4)**
- What about other probes? *Higher polynomials in **N***
- Anomalous Hall? **Yes**; but *cubic* in ***N***

$$\sigma_H^{xy} = a_3 N_y \left(3N_x^2 - N_y^2\right) + \ldots$$

Temperature dependence should follow temperature dependence of *N cubed*



Spin-splitting is *g*-wave (hexadecapolar)

Summary

- Simple symmetry based framework to predict responses
- Web of connections between key signatures (multipoles, AHE, weak FM, spin-splitting) from symmetry irreps.
- **Most responses dictated by spatial irrep of Neel order**
  - Appears in quadrupole? hexadecapole? (etc.)

For details:

P. A. McClarty & **J. G. Rau**, Phys. Rev. Lett. **132**, 176702 (2024) ❦

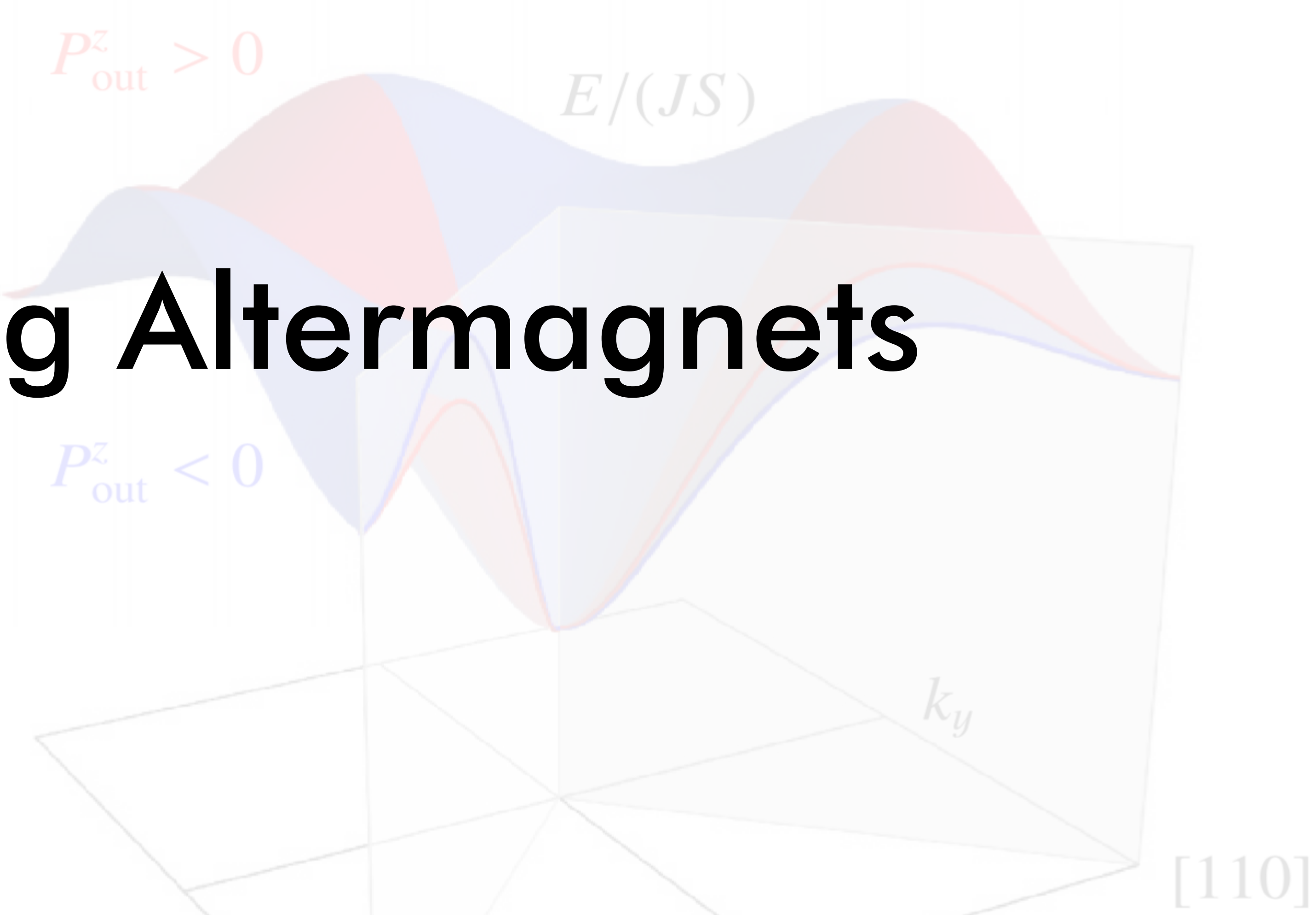
For extension to *all* point groups:

H. Schiff, P. A. McClarty, **J. G. Rau**, J. Romhanyi, arxiv:2412.18025

Take-aways

- Altermagnets *are* a new class of magnetic ordering, **in the non-relativistic limit**
  - *Not* once relativistic effects are included, *but many properties of non-relativistic limit are dictated by non-relativistic case*
- Properties of altermagnets are (mostly) determined by symmetries of the non-relativistic limit: **spin groups**
  - Symmetries where operations on space and on spin act *independently*
- *Some* of the properties of altermagnets can be understood from the paramagnetic phase, avoiding spin groups

Insulating Altermagnets



Mott insulating limit

- In limit of strong on-site interactions expect Mott insulating phase

$$-\sum_{ij} t_{ij} c_i^\dagger c_j + U \sum_i n_i^2$$

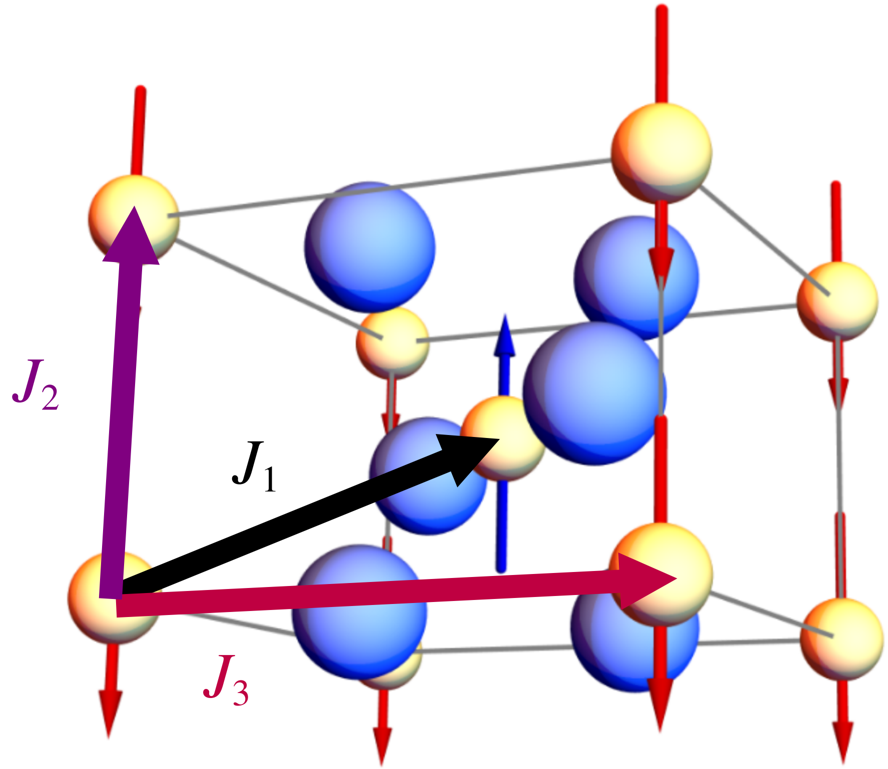
- Non-relativistic limit implies **isotropic** exchange interactions

$$\sum_{ij} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j \qquad J \sim \frac{t^2}{U}$$

- What do altermagnets look like in this limit?
  - What are their signatures?
  - How do they differ from conventional antiferromagnets?

Example: MnF_2

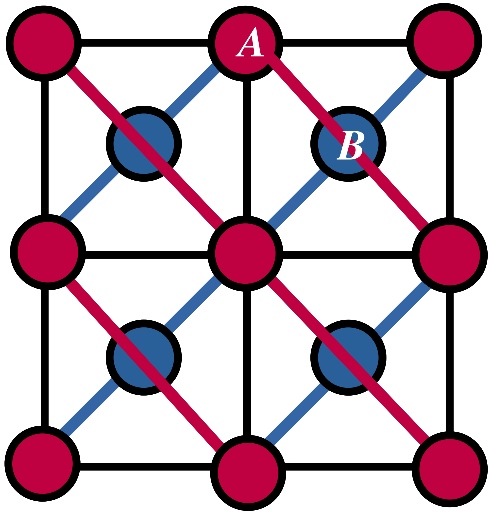
- Spin-only moment (S = 5/2) on Mn^{2+} sites
- Shortest exchange paths:
    - Nearest neighbour J_1
    - Next-nearest neighbour J_2, Third NN J_3
- *Model* is more symmetric than lattice – *"accidental" translation symmetry*
- Simplest model *still* looks like conventional antiferromagnet



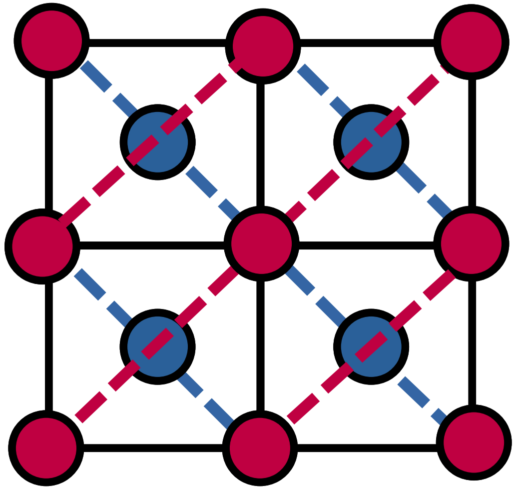
- “Altermagnetic” symmetry only realized past 3rd NN
- Shortest is diagonal intra-sublattice exchange along [110]

Common feature: Exchanges that realize that have altermagnetic character are often longer-range

- Longer range couplings are often *small* in Mott insulators
    - *How small* depends on details orbitals, ligands, etc



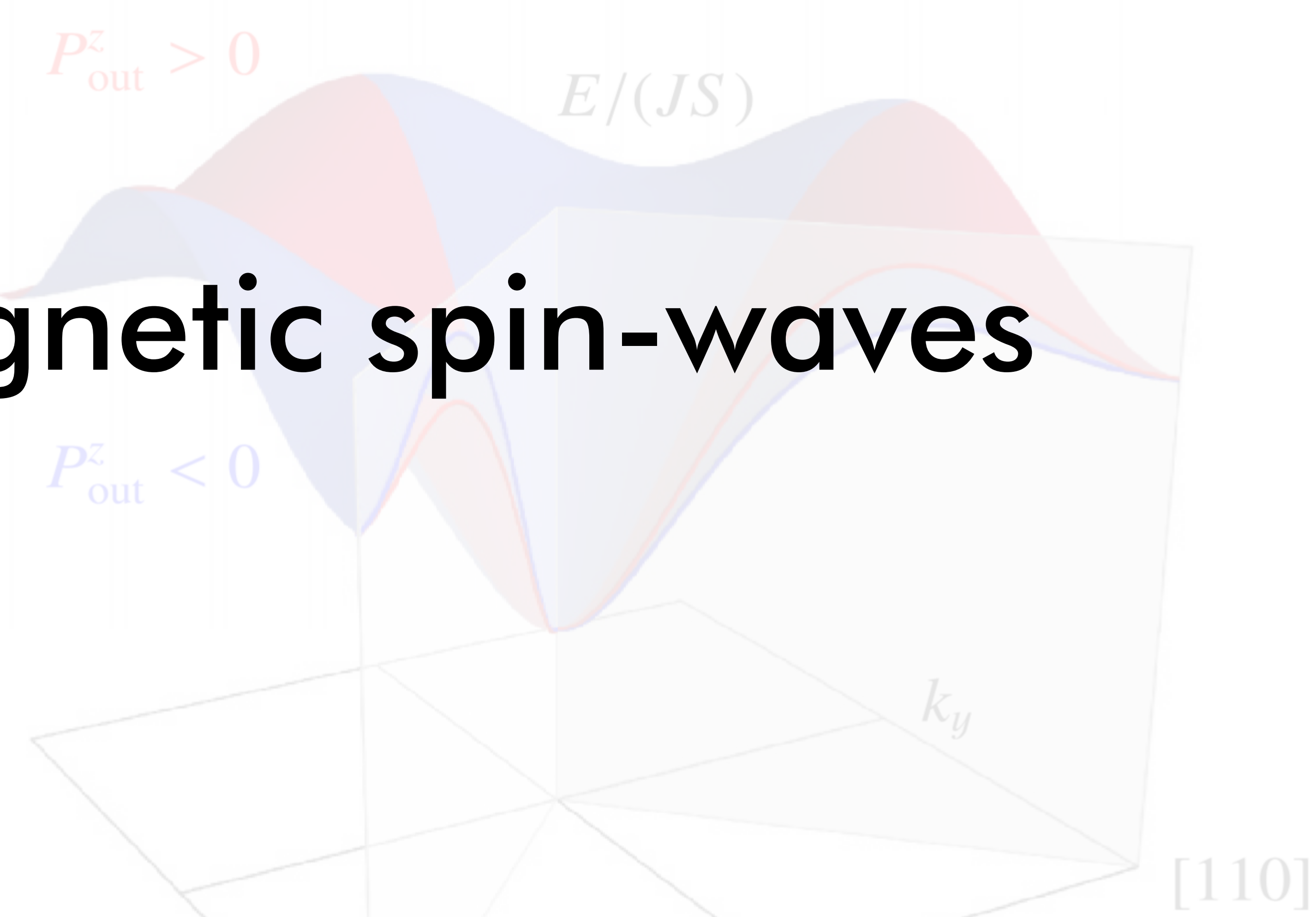
J_{110}



J'_{110}

All *altermagnetic signatures will depend on this difference*

Altermagnetic spin-waves



Holstein-Primakoff Expansion

- Bosonic representation of spin operators with respect to classical ground state

Transverse part *Longitudinal part*

$$\boldsymbol{S}_r = \sqrt{S}\left[\left(1-\frac{n_r}{2S}\right)^{1/2} a_r\hat{\boldsymbol{e}}_{r,-} + a_r^\dagger\left(1-\frac{n_r}{2S}\right)^{1/2}\hat{\boldsymbol{e}}_{r,+}\right] + (S-n_r)\hat{\boldsymbol{e}}_{r,0}$$

- where we have that $[a_r, a_{r'}^\dagger] = \delta_{rr'}$ and $n_r \equiv a_r^\dagger a_r$
- Two sublattice altermagnet, align along z-axis

$$\hat{z}_A \equiv +\hat{z}$$
$$\hat{z}_B \equiv -\hat{z}$$

- Semi-classical expansion, in the large S limit when $n_r/(2S)$ is small

Linear Spin-Wave Theory

- Plug this back into Hamiltonian and keep only leading terms in $1/S$

$$H = S^2 E_0 + S H_2 + \ldots$$

- Constant part at $O(S^2)$ is classical energy

$$E_0 = \frac{1}{2} \sum_{rr'} \hat{\boldsymbol{z}}_r^{\intercal} \boldsymbol{J}_{rr'} \hat{\boldsymbol{z}}_{r'}$$

- Quadratic boson Hamiltonian at O(S)

$$S H_2 = \sum_{rr'} \left[A_{rr'} a_r^{\dagger} a_{r'} + \frac{1}{2!} \left(B_{rr'} a_r^{\dagger} a_{r'}^{\dagger} + \text{h.c.} \right) \right]$$

"Hopping" like terms

"Pairing" like terms

Altermagnetic magnons

- What do you expect? **Colinear with U(1) symmetry**

$$a_{\boldsymbol{k},\mathsf{A}} \to e^{+i\theta} a_{\boldsymbol{k},\mathsf{A}}, \qquad a_{\boldsymbol{k},\mathsf{B}} \to e^{-i\theta} a_{\boldsymbol{k},\mathsf{B}}.$$

"up" sublattice *"down" sublattice*

- At the level of linear spin waves, strongly constrains possible coupling terms:

$$\sum_{\boldsymbol{k}} \left[A^{\mathsf{A}}_{\boldsymbol{k}} a^\dagger_{\boldsymbol{k},\mathsf{A}} a_{\boldsymbol{k},\mathsf{A}} + A^{\mathsf{B}}_{\boldsymbol{k}} a^\dagger_{\boldsymbol{k},\mathsf{B}} a_{\boldsymbol{k},\mathsf{B}} + \left(B^{\mathsf{AB}}_{\boldsymbol{k}} a^\dagger_{\boldsymbol{k},\mathsf{A}} a^\dagger_{-\boldsymbol{k},\mathsf{B}} + \text{h.c.} \right) \right]$$

*From **inter**-sublattice exchange* *From **intra**-sublattice exchange*

Crucially: Symmetry *does not require* that $A^{\mathsf{A}}_{\boldsymbol{k}} = A^{\mathsf{B}}_{\boldsymbol{k}}$

Altermagnetic magnons (cont.)

- Usual to write intra-sublattice parts as:

Altermagnetic part

$$A_{\boldsymbol{k}}^{\mathrm{A}} = A_{\boldsymbol{k}} + \delta A_{\boldsymbol{k}}/2$$
$$A_{\boldsymbol{k}}^{\mathrm{B}} = A_{\boldsymbol{k}} - \delta A_{\boldsymbol{k}}/2$$

$$\delta A_{\boldsymbol{k}} = 0$$ *Simple AFM*

R is symm. operation

$$\delta A_{R(\boldsymbol{k})} = -\delta A_{\boldsymbol{k}}$$ *Altermagnet*

- Spin-wave spectrum is just **shifted**

AFM would have degenerate bands

$$\epsilon_{\boldsymbol{k}} = \sqrt{A_{\boldsymbol{k}}^2 - B_{\boldsymbol{k}}^2} \pm \frac{1}{2}\delta A_{\boldsymbol{k}}$$

Eigenvectors unaffected by splitting

- **Altermagnets have anisotropic splitting of magnon bands**

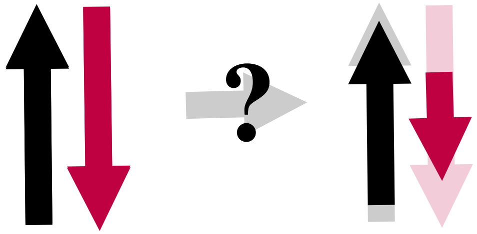
Aside: **Absence of piezomagnetism**

- Let's calculate the piezomagnetic response
  - *Still* colinear and two-sublattice order
- Magnetization along Neel vector, look at *difference* in sublattice magnetization

$$\delta M_A^z = \sum_{\boldsymbol{k}} \langle a_{\boldsymbol{k}A}^\dagger a_{\boldsymbol{k}A} \rangle = \sum_{\boldsymbol{k}} v_{\boldsymbol{k}}^2$$

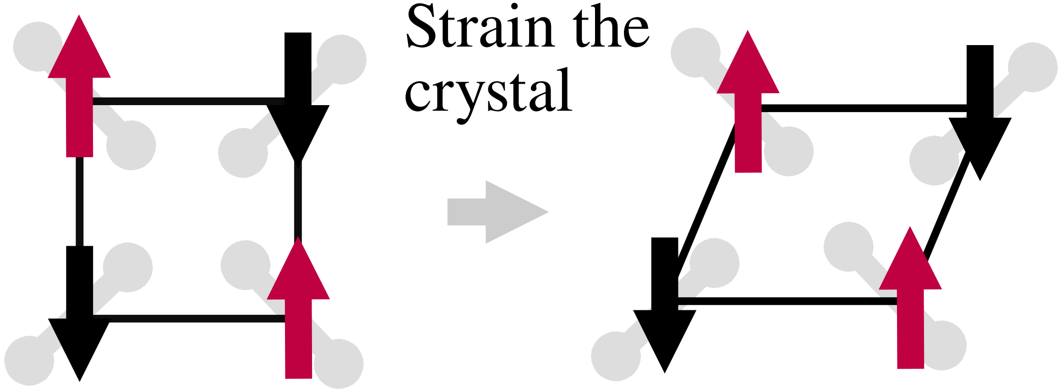
$$-\delta M_B^z = \sum_{\boldsymbol{k}} \langle a_{\boldsymbol{k}B}^\dagger a_{\boldsymbol{k}B} \rangle = \sum_{\boldsymbol{k}} v_{\boldsymbol{k}}^2$$

$$\delta A_{R(\boldsymbol{k})} \neq -\delta A_{\boldsymbol{k}}$$

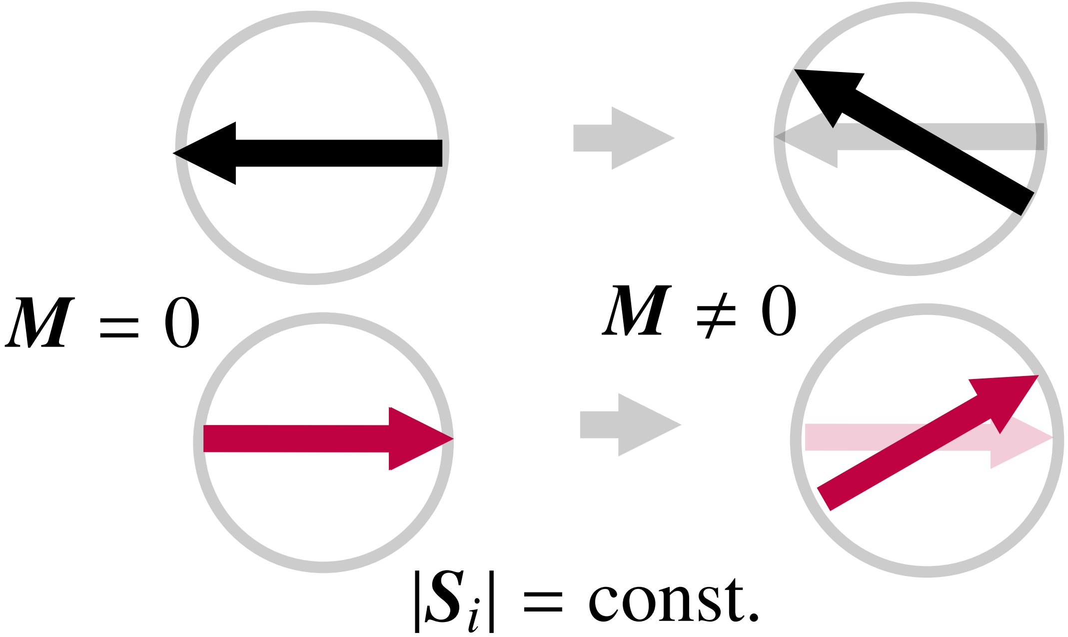


Symmetry not longer relates A & B

$$\delta M^z = \delta M_A^z + \delta M_B^z = 0$$



- In insulating antiferromagnet strain must induce *spontaneous breaking of U(1) symmetry* **immediately** to give piezomagnetism



Must *tilt* to induce finite $\boldsymbol{M}$ (gapped mode)

**Metallic case doesn't have hard constraint on spin length*

Luttinger's Theorem

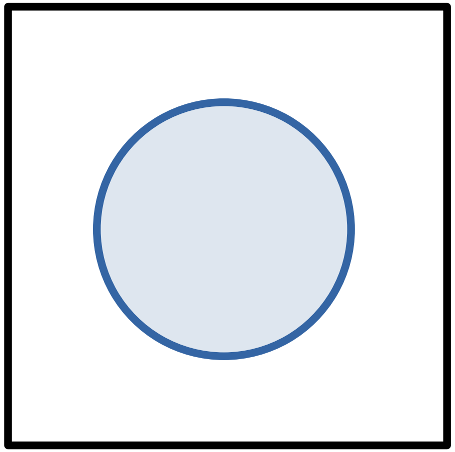
- Exact relation between density of electrons and Fermi surface volume

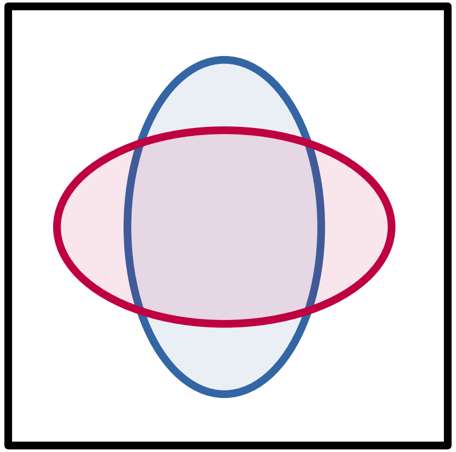
$$n = \frac{V_F}{V_{\mathrm{BZ}}} + \lfloor n \rfloor$$

- *Independent of spatial symmetry*
- Apply to up and down spins separately

$$n_\uparrow - n_\downarrow = \left(\frac{V_{F,\uparrow} - V_{F,\downarrow}}{V_{\mathrm{BZ}}} \right) + \lfloor n_\uparrow \rfloor - \lfloor n_\downarrow \rfloor$$

… assuming U(1) spin-rotation symmetry about z





Luttinger's theorem

$$\lfloor n_\uparrow \rfloor + \lfloor n_\downarrow \rfloor = N_c$$

- How to connect this Mott insulating* case?

$z_i = \pm 1$

$$H(\lambda) = (1-\lambda)H - \lambda \sum_i z_i S_i^z$$

* More direct argument for localized case via Oshikawa's theorem

- Assume ground state of $H(0)$ is colinear AFM; at $H(1)$ itinerant

$$H(1) = -\frac{1}{2} \sum_i z_i c_i^\dagger \sigma_z c_i$$

- Luttinger's theorem then implies that since Fermi volumes are **zero**

$$m = g\mu_B(n_\uparrow - n_\downarrow) = 0, \pm g\mu_B, \ldots$$

Luttinger Ferrimagnets

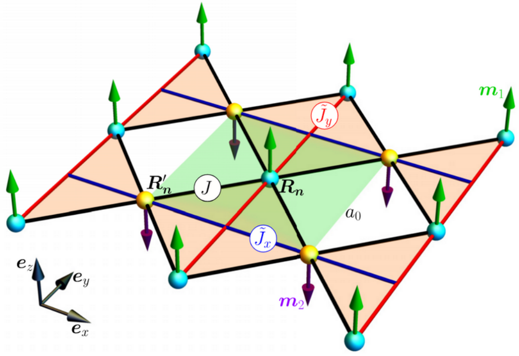
- *Different* mechanism of enforcing compensation
  - In *altermagnet* compensation protected by *symmetry*
  - … *but* Luttinger's theorem only allows discrete changes in magnetization per unit cell at $T = 0$
  - **"Luttinger ferrimagnet" –** compensated, but not due to symmetry
- *Immediate consequence:*
  - Strained insulating altermagnet (generically) is *Luttinger* compensated

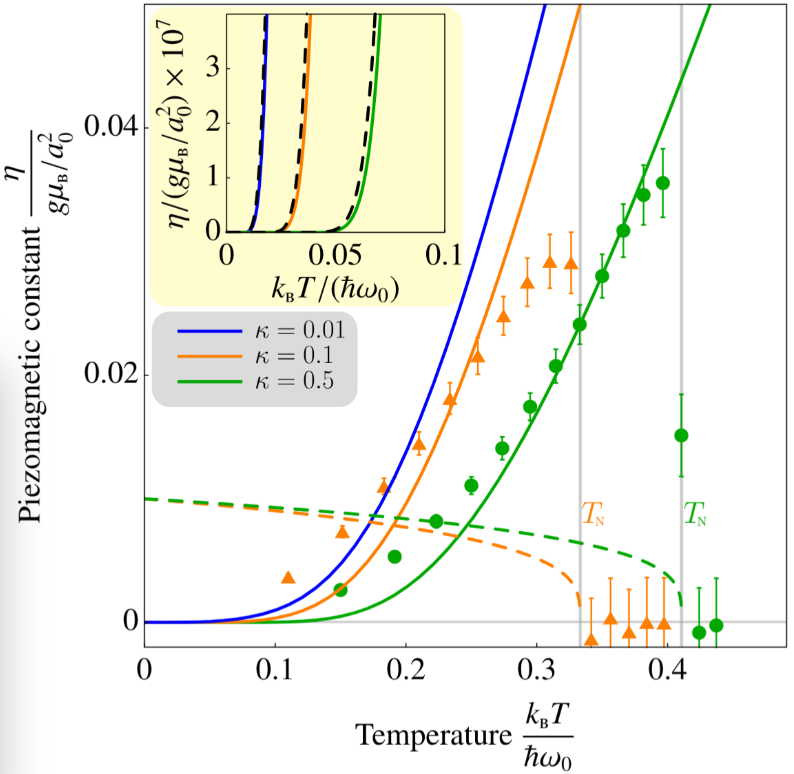
No piezomagnetism at $T = 0$ in insulating case

- Can show explicitly using spin-wave theory

$$\boldsymbol{M} = \sum_{\mu\nu} \boldsymbol{C}_{\mu\nu}\epsilon_{\mu\nu}$$

- Piezomagnetic coefficient vanishes algebraically in temperature ~ T^n
- Also appears in classical models





PHYSICAL REVIEW B **110**, 144421 (2024)

Editors' Suggestion

Fluctuation-induced piezomagnetism in local moment altermagnets

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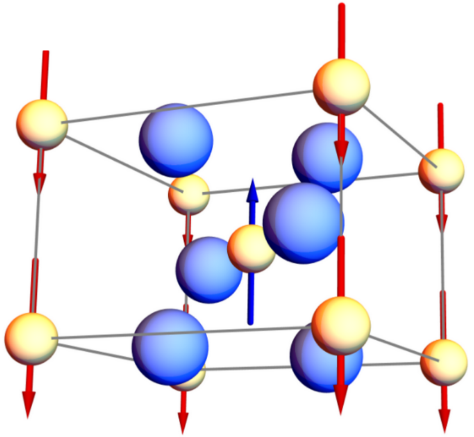
Example: *MnF₂*

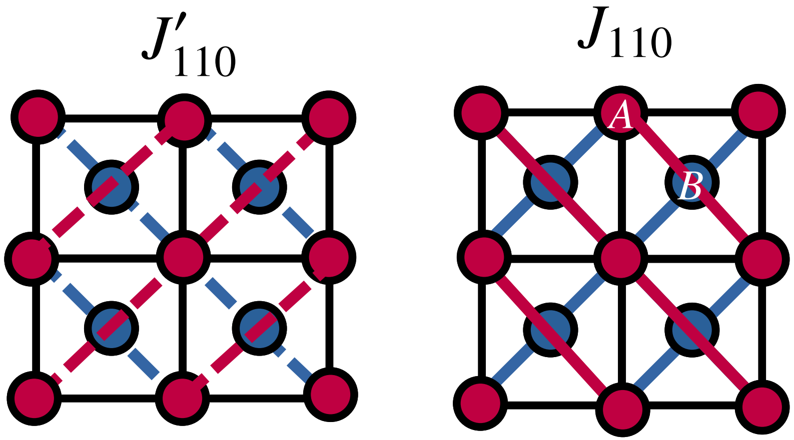


- Minimal model: NN coupling (J) and longer-range “altermagnetic” exchanges

$$A_{\boldsymbol{k}} = 8SJ + 2S(J_{110} + J'_{110})[\cos(ak_x)\cos(ak_y) - 1]$$

$$\delta A_{\boldsymbol{k}} = 4S(J_{110} - J'_{110})\sin(ak_x)\sin(ak_y),$$

$$B_{\boldsymbol{k}} = -8SJ\gamma_{\boldsymbol{k}}.$$



- Finite altermagnetic splitting *if* these two exchange paths differ in magnitude

Gohlke *et al*. Phys. Rev. Lett. **131**, 186702 (2024)

Example: MnF_2

- Low-energy, long-wavelength physics (Goldstone mode)

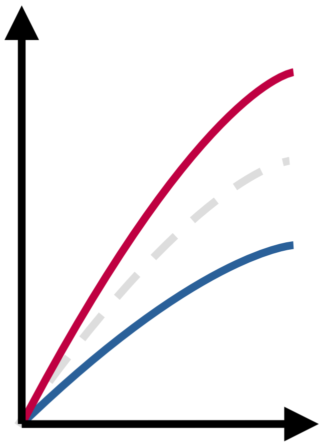
$$A_{\boldsymbol{k}} \approx 8SJ + S(J_{110} + J'_{110})a^2|\boldsymbol{k}|^2 \equiv A_0 + \alpha|\boldsymbol{k}|^2$$

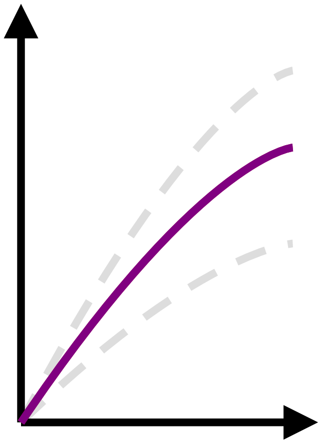
$$\delta A_{\boldsymbol{k}} \approx 4S(J_{110} - J'_{110})a^2 k_x k_y \equiv 2\eta k_x k_y$$

$$B_{\boldsymbol{k}} \approx -A_0 + \beta|\boldsymbol{k}|^2$$

- Goldstone mode, still linear but split

$$\epsilon_{\boldsymbol{k}} \approx \sqrt{2A_0(\alpha+\beta)}|\boldsymbol{k}| \pm \eta k_x k_y = v|\boldsymbol{k}| \pm \eta k_x k_y$$



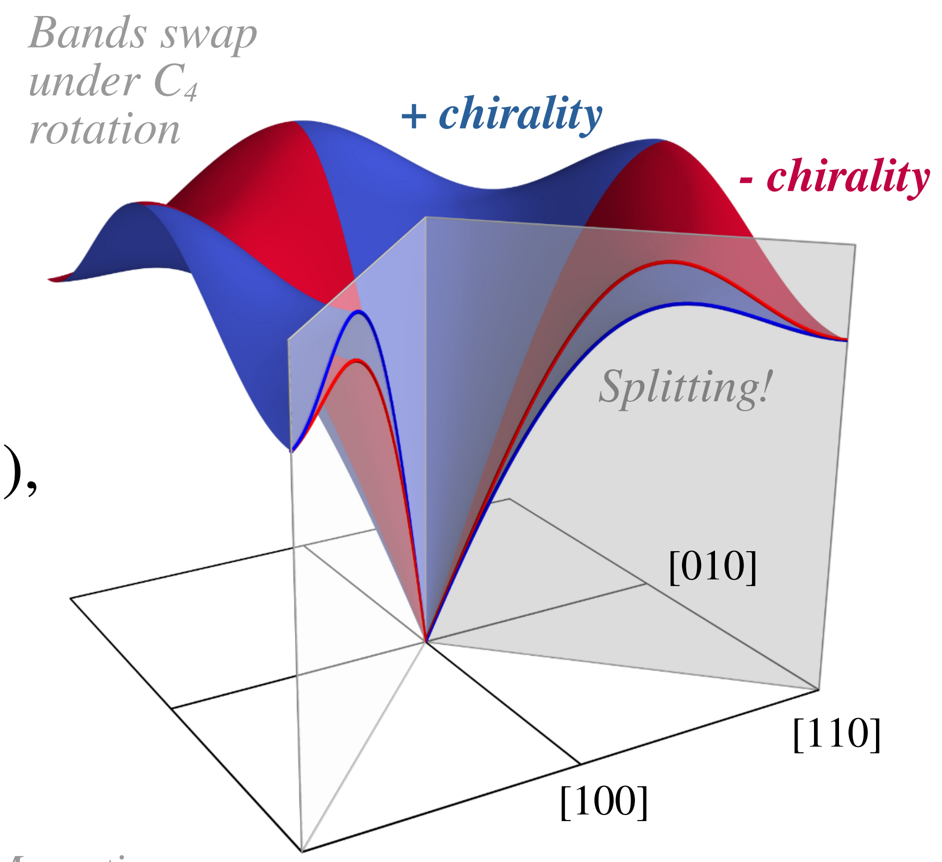


Example: MnF_2

- Pattern of splitting of altermagnetic magnons mirror that in the electronic band structure

$$\delta A_{\boldsymbol{k}} = 4S\,(J_{110} - J'_{110})\sin(ak_x)\sin(ak_y),$$

- Splitting *vanishes* along many high-symmetry directions
  - *Both k_x and k_y non-zero*
- ***Each band has a well-defined "chirality"***

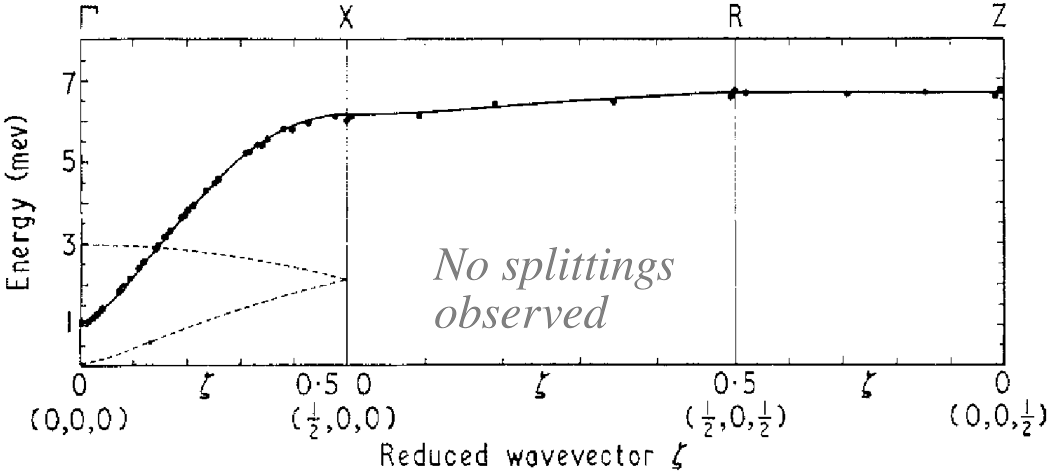


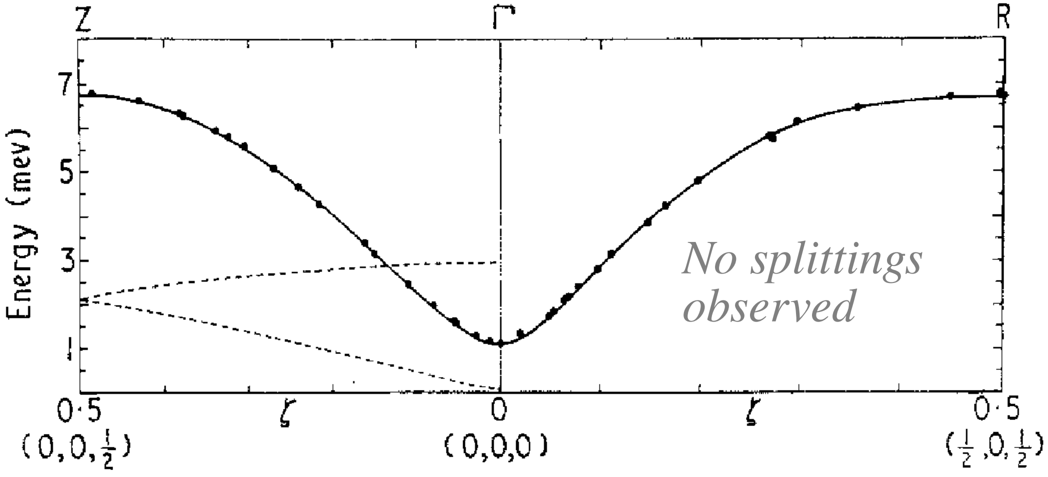
"Magnetic moment" of magnon

Gohlke *et al*. Phys. Rev. Lett. **131**, 186702 (2023), Smejkal *et al*, Phys. Rev. Lett. 131, 256703 (2023)

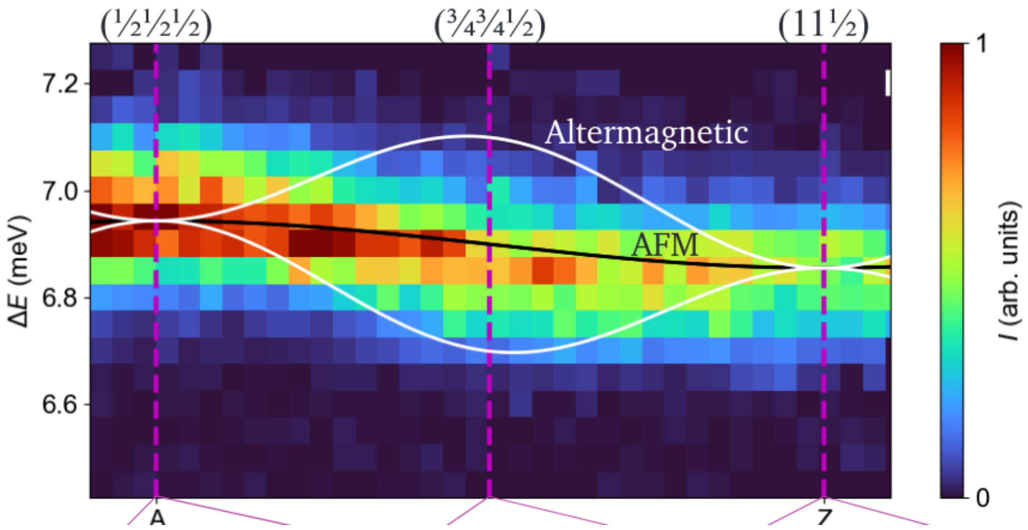
Example: MnF_2

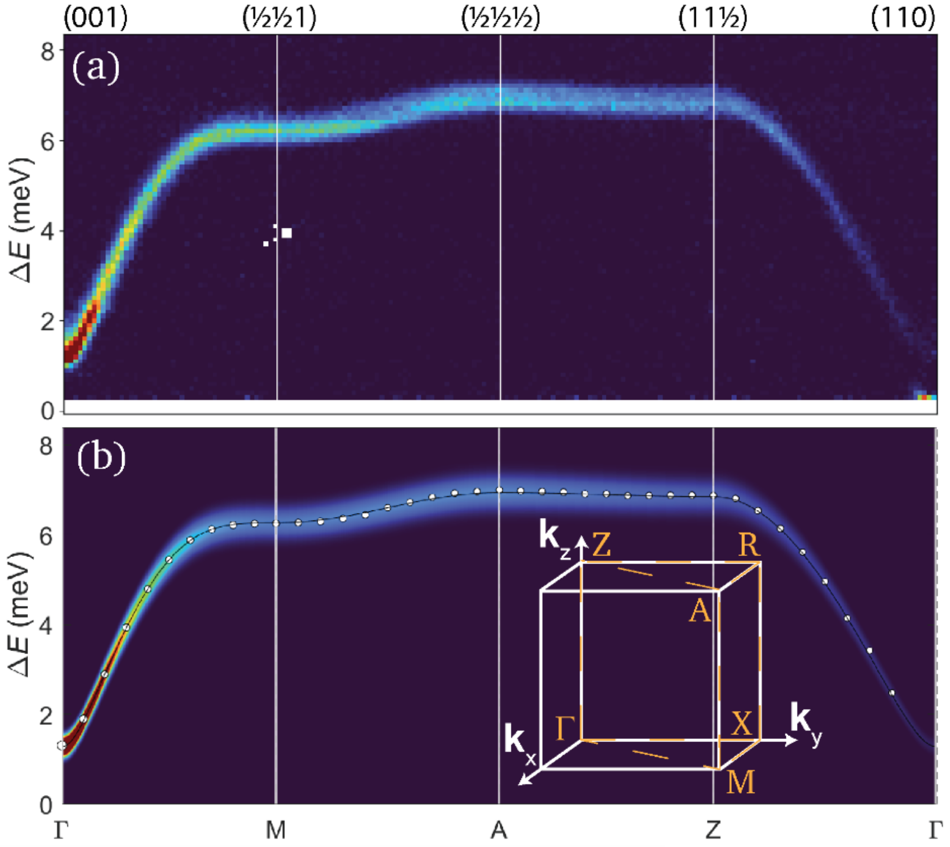
- Studied since the 1960s and 1970s!
  - Lots of data & analysis
- No evidence for splitting
  - **All data along high-symmetry lines where it vanishes**
- Should be likely be revisited ...





Nikotin *et al*. J Phys. C, **2**, 1168 (1969)





… no observable splitting to a resolution of ~ 0.2 meV or so

PHYSICAL REVIEW LETTERS **134**, 226702 (2025)

Absence of Altermagnetic Magnon Band Splitting in MnF_2

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(Received 6 December 2024; accepted 28 April 2025; published 4 June 2025)

Example: *MnTe*

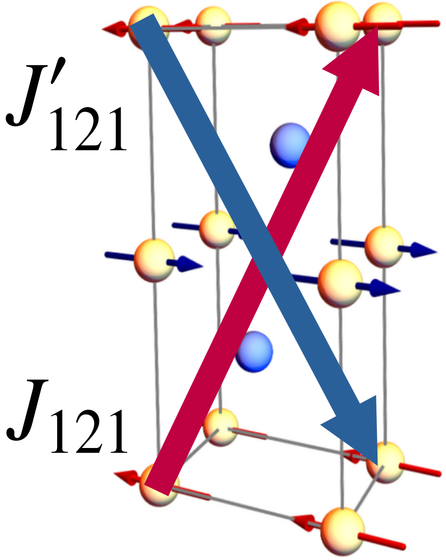
- … try another candidate material
- Minimal model: Nearest-neighbour exchange and the long-range (>10th NN) altermagnetic exchange

$$A_{\boldsymbol{k}} = 2SJ + S(J_{121} + J'_{121})(\gamma_{\boldsymbol{k}} + \gamma'_{\boldsymbol{k}} - 6),$$

$$\delta A_{\boldsymbol{k}} = 2S(J_{121} - J'_{121})(\gamma_{\boldsymbol{k}} - \gamma'_{\boldsymbol{k}})$$

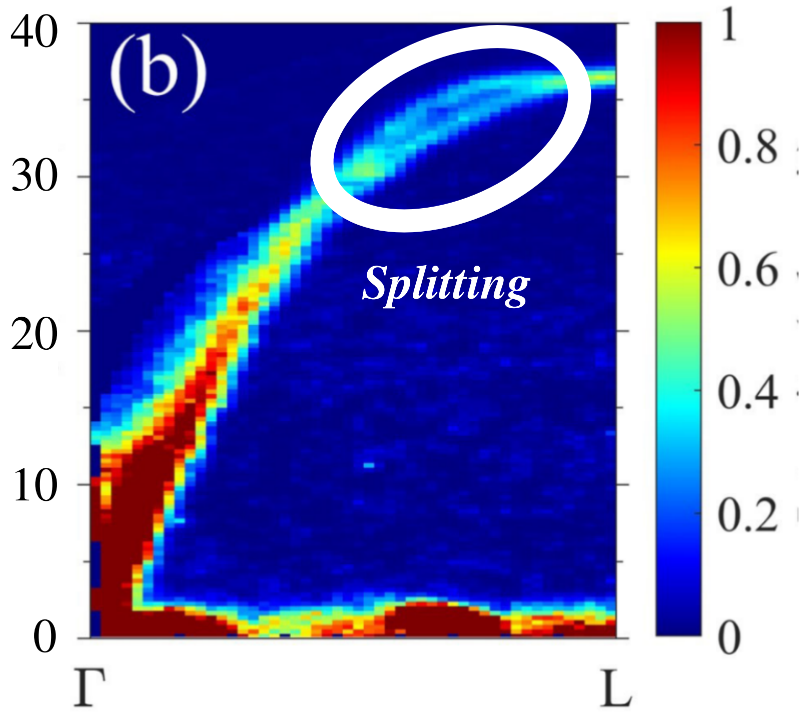
$$B_{\boldsymbol{k}} = 2SJ\cos(ck_z/2)$$

- *g*-wave splitting due to hexagonal symmetry

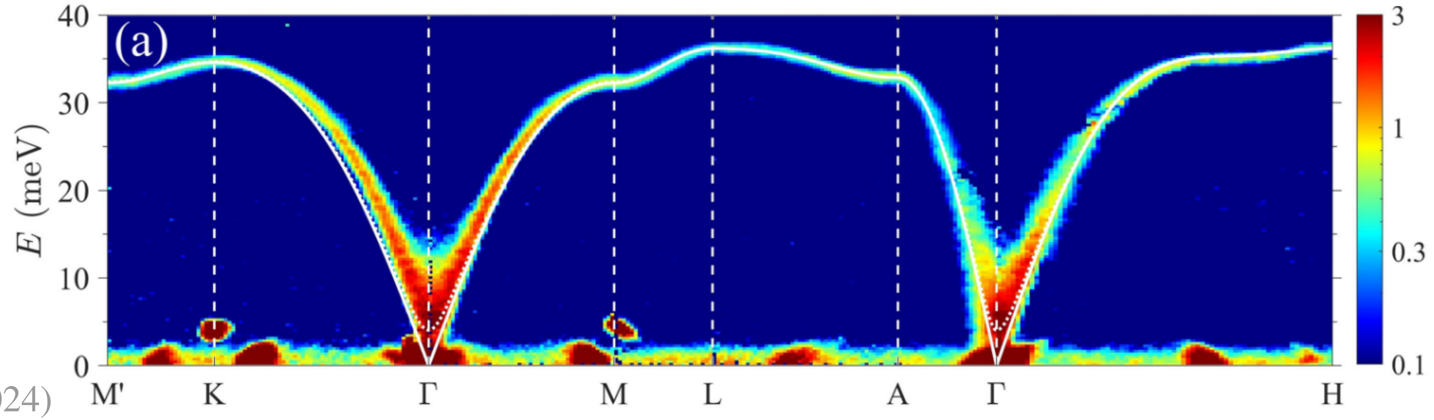


Example: *MnTe*

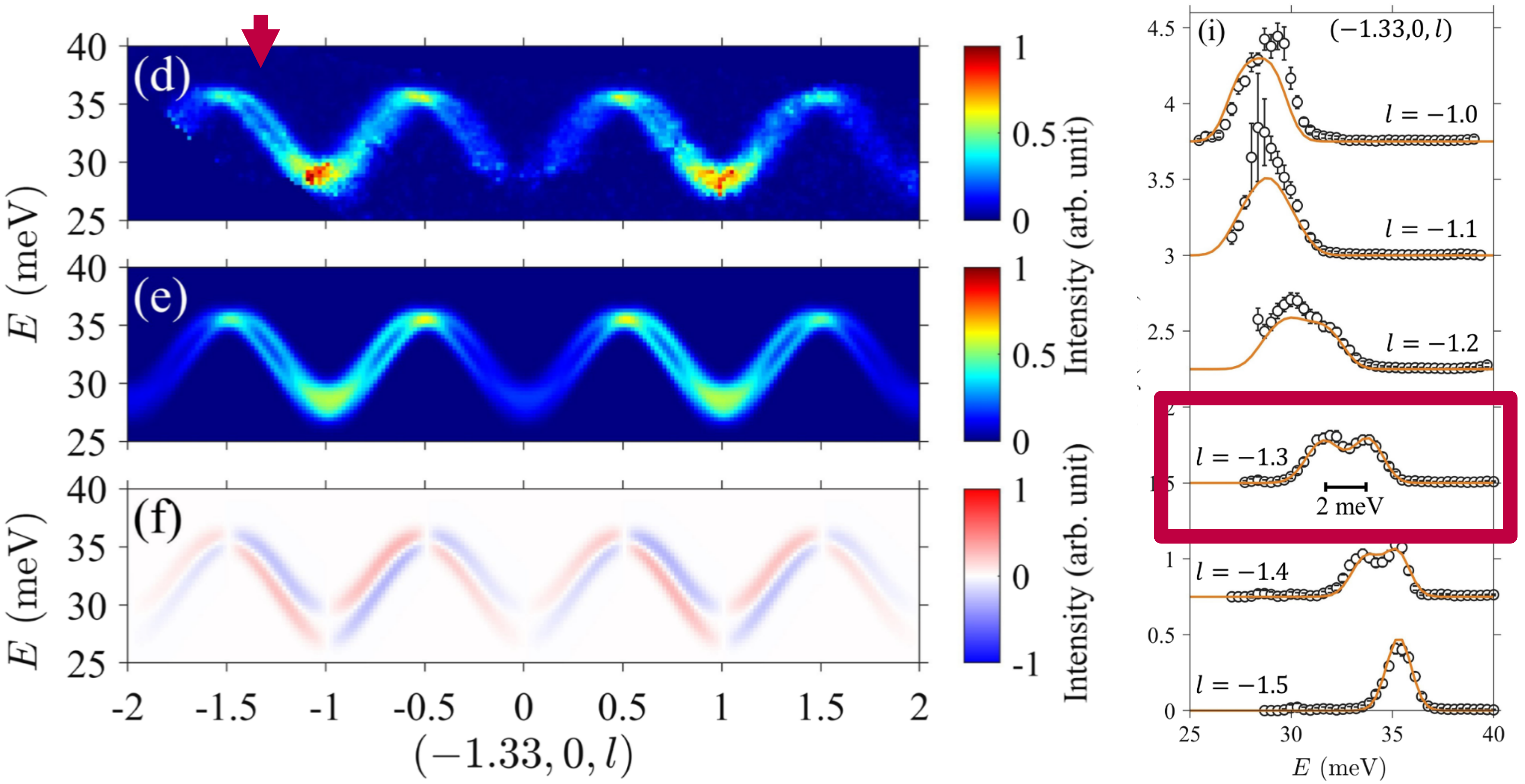
- **Magnon splitting observed experimentally!**
- Small, but visible along (expected) cuts in momentum space



What is the scale of altermagnetic exchange?

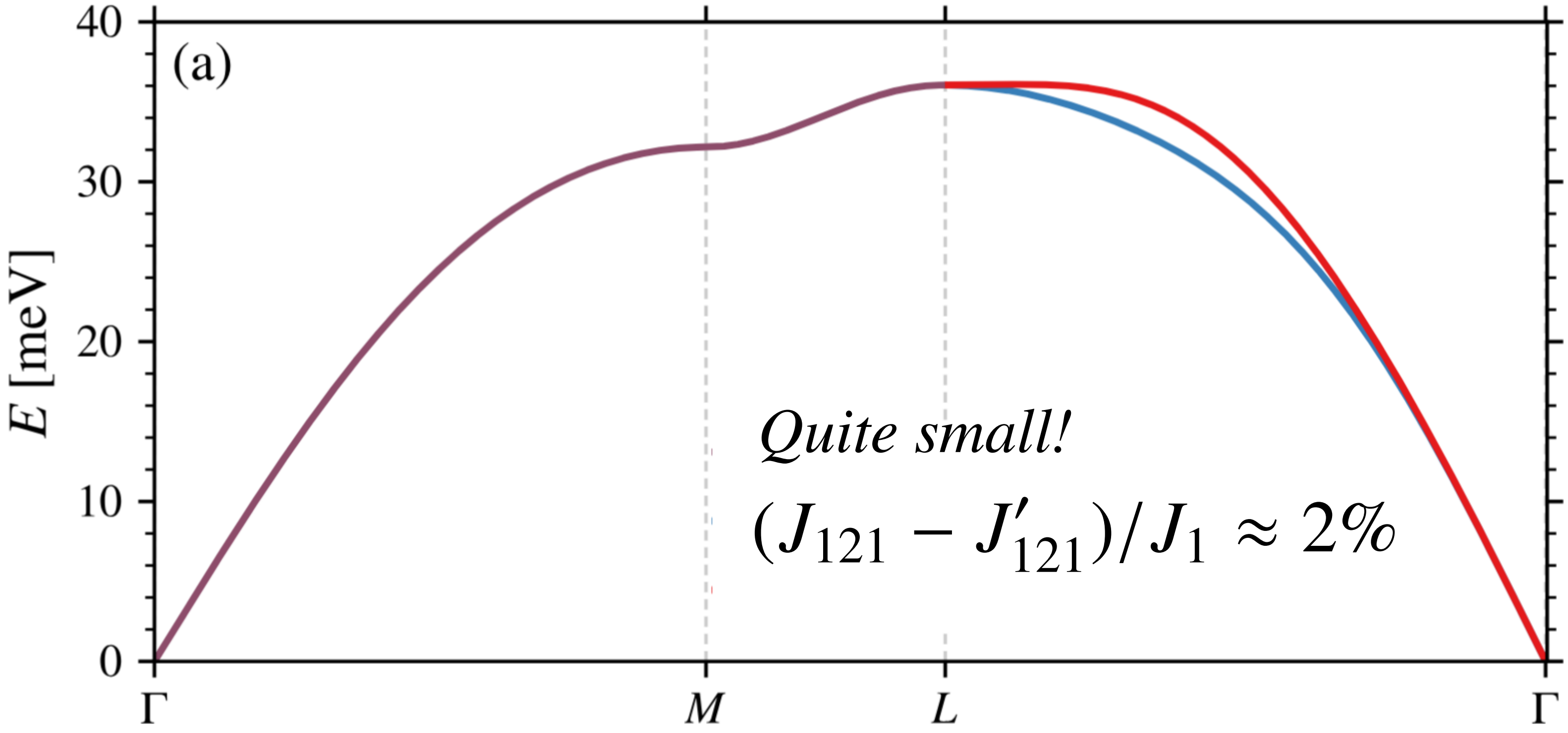


Liu *et al*, Phys. Rev. Lett **133**, 156702 (2024)



(d)
(e)
(f)
E (meV)
Intensity (arb. unit)
(−1.33, 0, l)
(i)
(−1.33,0, l)
l = −1.0
l = −1.1
l = −1.2
l = −1.3
l = −1.4
l = −1.5
2 meV
E (meV)

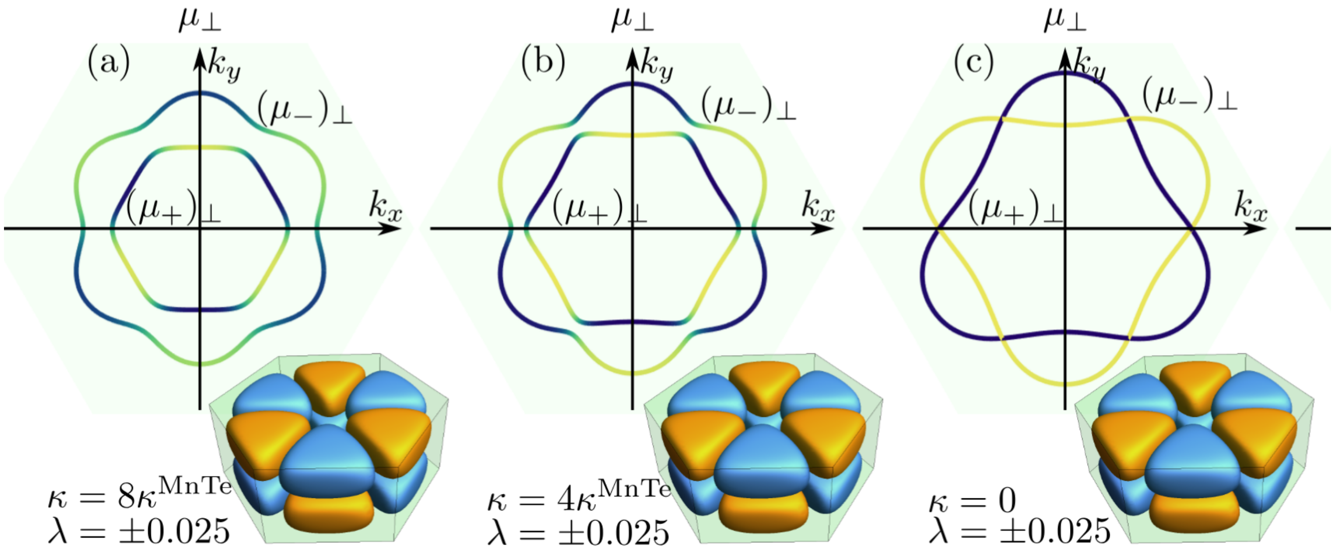
Example: *MnTe*



$J_1 = 3.99$ meV, $J_{121} = 0.068$ meV, $J'_{121} = -0.022$ meV

How do we know it is not spin-orbit?

Effect of spin-orbit coupling



Arxiv:2504.05241

- Spin-orbit lifts degeneracies and mixes chiralities
- “Line nodes” are split
- Similar to spin-splitting in the metallic case

Chiral magnetic excitations and domain textures of g-wave altermagnets

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[1] Institute for Theoretical Solid State Physics, Leibniz Institute for Solid

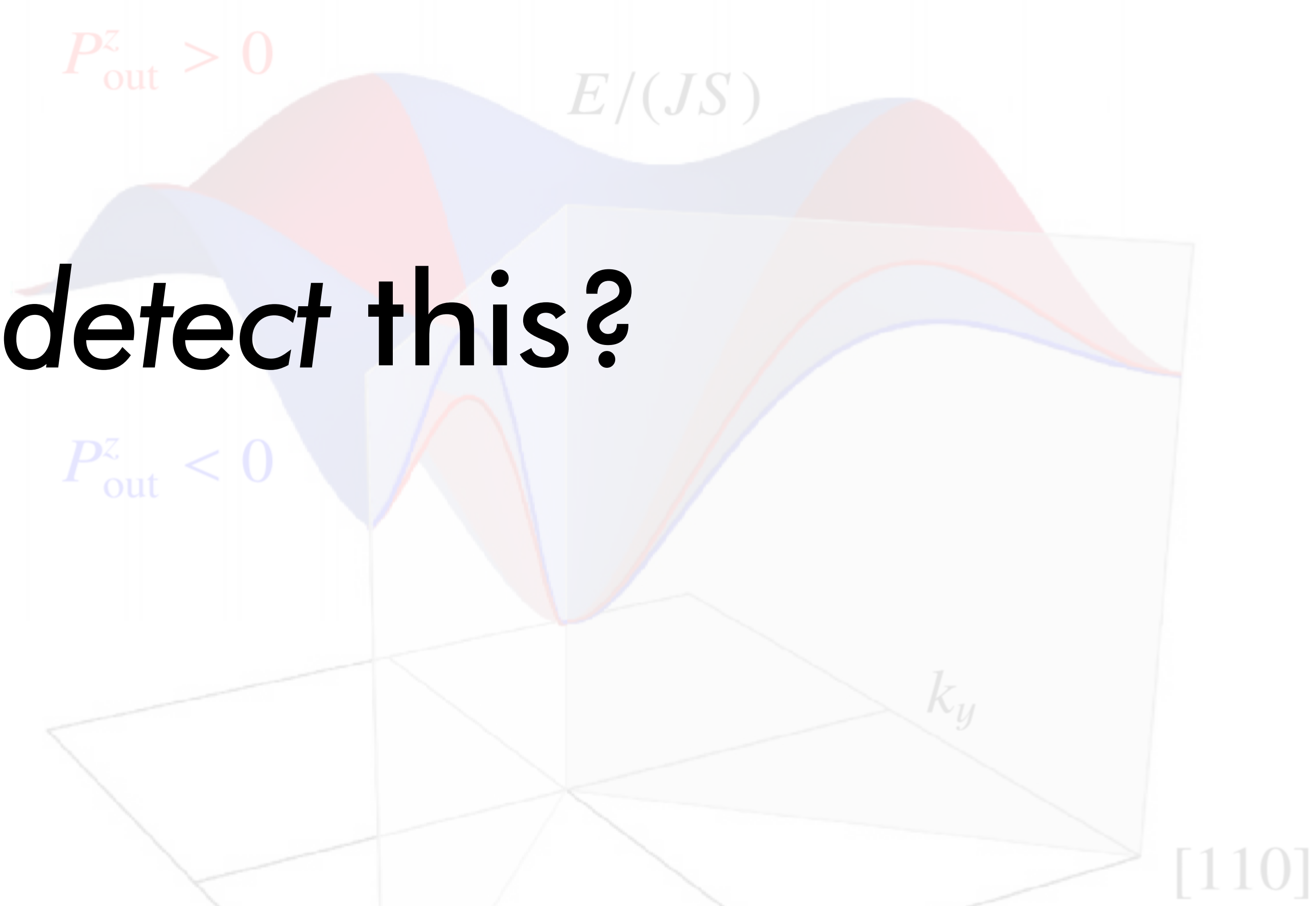
Practical Challenges

- What determines if this splitting is visible?
  - Altermagnetic exchange difference *always* there

$$\delta J \gtrsim D, \Gamma, \ldots$$

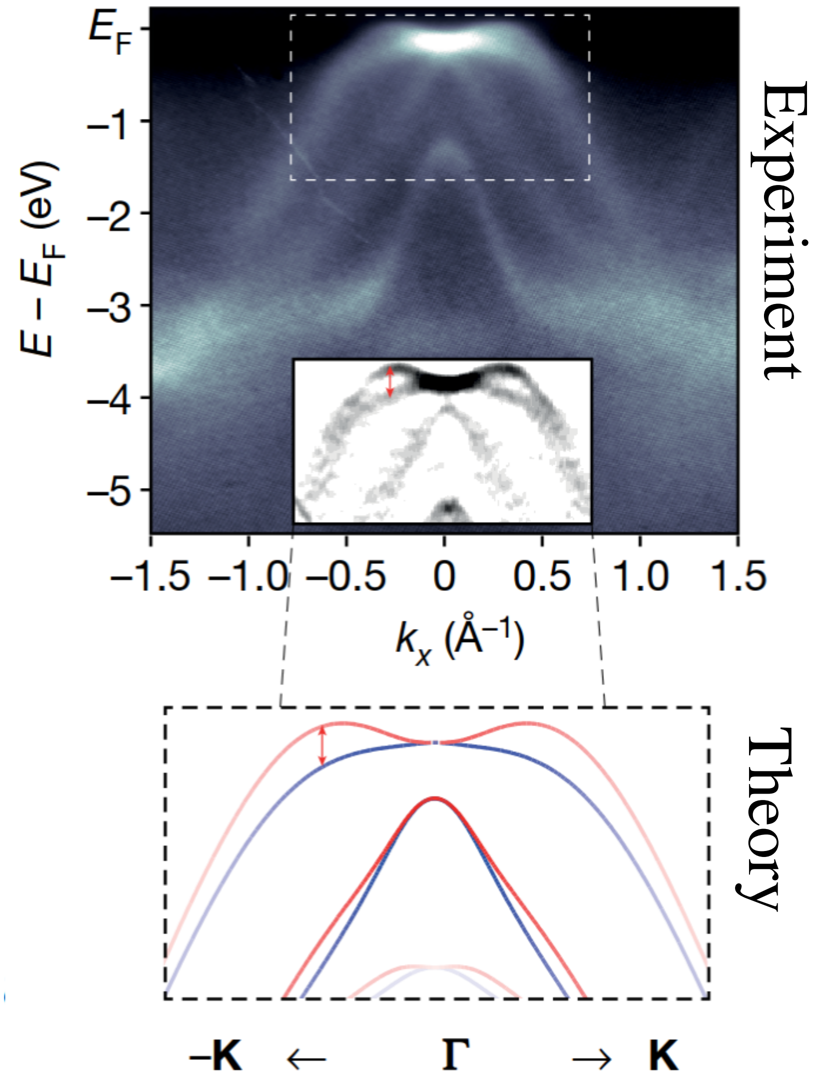
- **Question:** Are the altermagnetic exchanges competitive with the anisotropic exchange?
  - Compare the splittings at band crossings/zone boundary to the altermagnetic splitting
- Exacerbated by smallness of δJ due to long-range

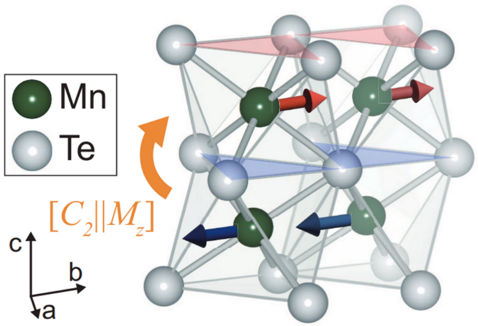
Can we *detect* this?



Spin-split bands

- *Some* experimental data supporting spin-split bands in candidate altermagnets
- **Example:** MnTe
- Directly compare bands using ARPES in the paramagnetic and ordered phases





Spin-waves?

- Can we see a signature of altermagnetism in the spin-waves of an altermagnet?
  - *Usual tool:* Inelastic neutron scattering
- Measures moment-moment correlation function

$$\left(\frac{d^2\sigma}{d\Omega d\omega}\right) \propto \int dt e^{-i\omega t} \langle \boldsymbol{M}^{\perp}_{-\boldsymbol{k}} \cdot \boldsymbol{M}^{\perp}_{\boldsymbol{k}}(t) \rangle \qquad M^{\perp}_{k} = \hat{k} \times (M_k \times \hat{k})$$

$$\propto \sum_n \mathcal{W}_{\boldsymbol{k},n} \delta(\omega - \epsilon_{\boldsymbol{k},n}).$$

Intensity of mode ($\mathcal{W}_{\boldsymbol{k},n}$); *Spin-wave energies* ($\epsilon_{\boldsymbol{k},n}$)

How do observe chiral magnon modes?

Polarized Neutrons

- Polarized neutron scattering allows a *direct probe of altermagnetic character of magnon bands*

Usual unpolarized part — *Polarized neutron beam* — *Access anti-symmetric part*

$$\left(\frac{d^2\sigma}{d\Omega d\omega}\right) \propto \int dt e^{-i\omega t}\left[\langle \boldsymbol{M}^{\perp}_{-\boldsymbol{k}} \cdot \boldsymbol{M}^{\perp}_{\boldsymbol{k}}(t)\rangle + i\boldsymbol{P}_{\text{in}} \cdot \langle \boldsymbol{M}^{\perp}_{-\boldsymbol{k}} \times \boldsymbol{M}^{\perp}_{\boldsymbol{k}}(t)\rangle\right]$$

Intensity

- Also works when send in *unpolarized* neutrons and look at *outgoing* polarization

Accesses same anti-symmetric part

Polarization of neutrons

$$\boldsymbol{P}_{\text{out}} = \left(\frac{d^2\sigma}{d\Omega d\omega}\right)^{-1}_{\boldsymbol{P}_{\text{in}}=0} \int dt e^{-i\omega t}\left[-i\langle \boldsymbol{M}^{\perp}_{-\boldsymbol{k}} \times \boldsymbol{M}^{\perp}_{\boldsymbol{k}}(t)\rangle\right]$$

Brown Proc. Roy. Soc **442** 147 (1969), Blume, Phys. Rev. **130**, 1670 (1963), Moon *et al*. Phys. Rev. **181**, 920 (1969), ...

Polarized Neutrons (cont.)

- For our simple altermagnet within linear spin-wave theory

One magnon intensity

$$\mathcal{W}_{\boldsymbol{k},n}(\boldsymbol{P}_{\text{in}}) = \left(\frac{1}{2}[1 + (\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{N}})^2] + (-1)^n(\boldsymbol{P}_{\text{in}} \cdot \hat{\boldsymbol{k}})(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{N}})\right) C_{\boldsymbol{k}}$$

Polarized part changes sign depending on band

- Can isolate this part by doing two experiments: one with ***P*** along +*N* and then along -*N* and looking at relative difference

$$\frac{\Delta \mathcal{W}_{\boldsymbol{k},n}}{\mathcal{W}_{\boldsymbol{k},n}(\boldsymbol{0})} = (-1)^n \left(\frac{2(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{N}})^2}{1 + (\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{N}})^2}\right)$$

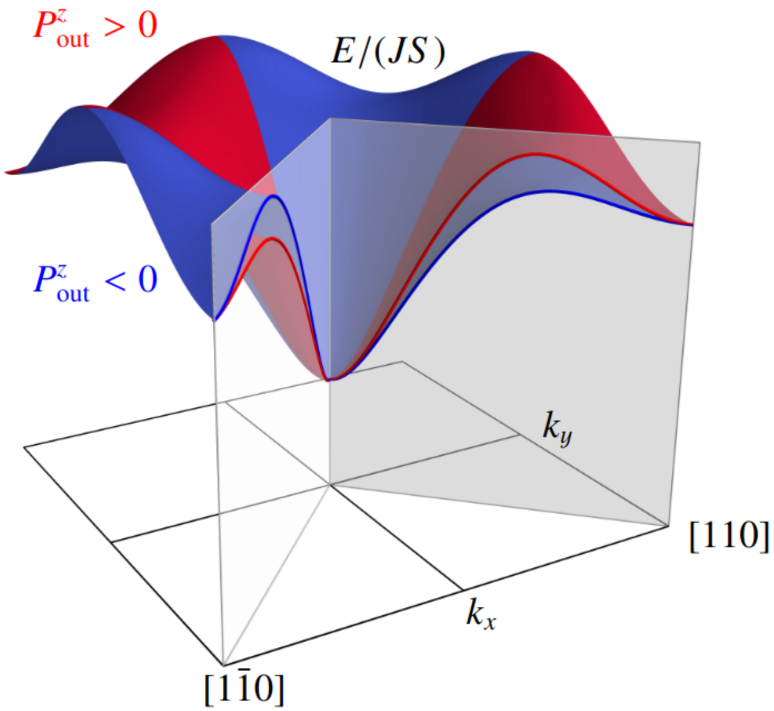
Directly detects chirality of magnon band

Polarized Neutrons (cont.)

- Identical argument for measuring polarization from unpolarized beam

$$\boldsymbol{P}_{\mathrm{out}}(\boldsymbol{k}, \epsilon_{\boldsymbol{k},n}) = -(-1)^n \left(\frac{2(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{N}})}{1 + (\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{N}})^2} \right) \hat{\boldsymbol{k}}.$$

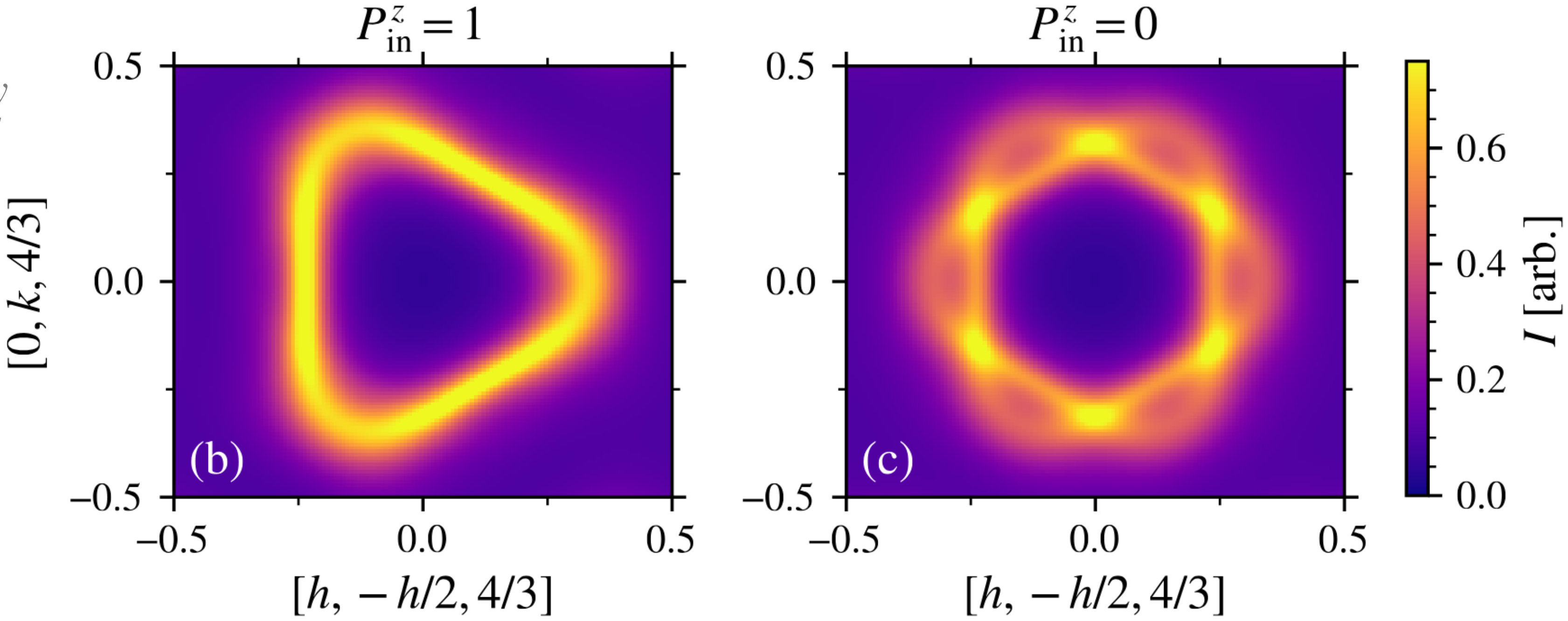
- Neutron polarization can directly image the **chirality splitting and spatial anisotropy** of the magnon bands



Example: *MnTe*

- Using a polarized incoming beam, you'd have a crossover from isotropy to anisotropy as the polarization increases

Single chirality selected



Stability and domains

- Since chiral bands are separated in energy **stable to small perturbations like spin-orbit coupling**
- *Stronger statement:* Perturbations (SOC or not) that are not altermagnetic in character make even/odd superpositions of chiral bands
  - **Expect non-altermagnetic splittings induce small chirality**
- Polarized neutrons (via nuclear-magnetic cross-term) can provide a way to probe domain imbalance in altermagnets

Vanishes in simple AFMs

$$\propto \left(\boldsymbol{N} \cdot \boldsymbol{P}_{\text{in}}^{\perp}\right) \mathrm{Re}\left[F_{\text{nuc}}(\boldsymbol{G}) F_{\text{mag}}^{*}(\boldsymbol{G})\right].$$

Non-zero in altermagnets

Domains

- To see this effect **need imbalance of time-reversed domains** (ideally single domain)
  - Prepare via field cooling (e.g. MnF_2*), electrical methods via spin-orbit torque, ...
- *Like in ferromagnets* polarized nuclear-magnetic cross-term probes domain imbalance in altermagnets

Vanishes in simple AFMs

$$\propto \left(\boldsymbol{N} \cdot \boldsymbol{P}_{\mathrm{in}}^{\perp}\right) \mathrm{Re}\left[F_{\mathrm{nuc}}(\boldsymbol{G}) F_{\mathrm{mag}}^{*}(\boldsymbol{G})\right].$$

Non-zero in altermagnets

- What method used, polarized neutrons can confirm presence of imbalance

*Alperin *et al.* Phys. Rev. Lett, **8**, 237 (1962)

Take-aways

- Magnetism of insulating altermagnets often has description in terms of local moment model
  - *Quirk:* Absence of piezomagnetism at $T = 0$ due length constraint on local moment (or Luttinger's theorem)
- **Signature of altermagnetic symmetry is anisotropically split *chiral* magnon bands**
  - Splitting is observable using inelastic neutron scattering
  - Polarization dependence reveals chirality

Polarized neutron signature:
McClarty, Gukasov, Rau, Physical Review B **111**, L060405 (2025)