The 230 Space Groups

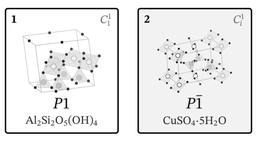
Arranged by crystal system and point group, in the order of the International Tables. Each tile pairs a space group with a crystal that has exactly its symmetry.

Key

- $\bar{1}$ Centrosymmetric — contains the inversion
- χ Sohncke — proper rotations only
- ↑ Polar — a direction every operation keeps
- ▢ Symmorphic — no screw axes or glide planes
- ⇄ Enantiomorphic — one of a mirror-image pair

Triclinic

$1\ \chi\uparrow$ $\quad \bar{1}\ \bar{\imath}$



Crystal Systems



Triclinic — *P*
no conditions



Monoclinic — *P*, *C*
$\alpha = \gamma = 90°$



Orthorhombic — *P*, *C*, *I*, *F*
$\alpha = \beta = \gamma = 90°$



Tetragonal — *P*, *I*
$a = b$; $\alpha = \beta = \gamma = 90°$



Trigonal — (*P*), *R*
$a = b$, $\gamma = 120°$;
or $a = b = c$, $\alpha = \beta = \gamma$



Hexagonal — *P*
$a = b$; $\alpha = \beta = 90°$, $\gamma = 120°$

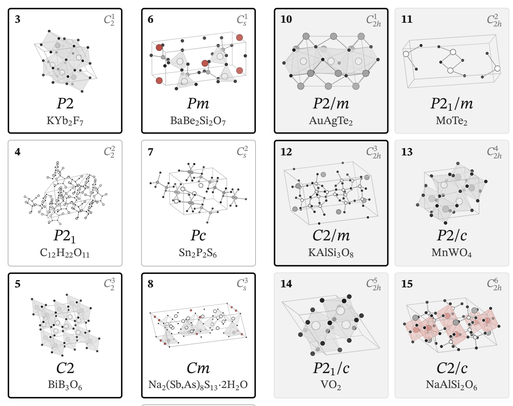


Cubic — *P*, *I*, *F*
$a = b = c$; $\alpha = \beta = \gamma = 90°$

With its centrings, each system gives the 14 Bravais lattices (trigonal's *P* is hexagonal's).

Monoclinic

$2\ \chi\uparrow$ $\quad m\uparrow$ $\quad 2/m\ \bar{\imath}$



↑ Pyroelectricity, ferroelectricity
▢ SHG, piezoelectricity (all but 432)
χ Chirality, optical activity

9 C_s^4 — Cc, $FeMo_2S_4$

1 Kaolinite, porcelain clay
2 $CuSO_4{\cdot}5H_2O$ copper sulfate
4 $C_{12}H_{22}O_{11}$ table sugar
5 BiB_3O_6 nonlinear optical crystal
11 $MoTe_2$ topological semimetal
12 $KAlSi_3O_8$ orthoclase feldspar
13 $MnWO_4$ multiferroic
14 VO_2 metal–insulator transition
15 $NaAlSi_2O_6$ jade

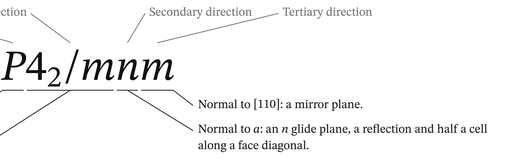
The 32 Point Groups

Triclinic		Monoclinic		Orthorhombic		Tetragonal		Trigonal		Hexagonal		Cubic	
1 C_1	1	2 C_2	3	222 D_2	9	4 C_4	6	3 C_3	4	6 C_6	6	23 T	5
$\bar{1}$ C_i	1	m C_s	4	$mm2$ C_{2v}	22	$\bar{4}$ S_4	2	$\bar{3}$ C_{3i}	2	$\bar{6}$ C_{3h}	1	$m\bar{3}$ T_h	7
		$2/m$ C_{2h}	6	mmm D_{2h}	28	$4/m$ C_{4h}	6	32 D_3	7	$6/m$ C_{6h}	2	432 O	8
						422 D_4	10	$3m$ C_{3v}	6	622 D_6	6	$\bar{4}3m$ T_d	6
						$4mm$ C_{4v}	12	$\bar{3}m$ D_{3d}	6	$6mm$ C_{6v}	4	$m\bar{3}m$ O_h	10
						$\bar{4}2m$ D_{2d}	12			$\bar{6}m2$ D_{3h}	4		
						$4/mmm$ D_{4h}	20			$6/mmm$ D_{6h}	4		

Each in Hermann–Mauguin and Schoenflies notation, with its number of space groups; shaded: centrosymmetric. First derived by Hessel (1830) and, independently, Gadolin (1867).

C_n is an n-fold rotation axis
D_n is C_n with n twofold axes $\perp$ it
S_n is an n-fold rotoreflection axis
T, O are the rotations of a tetrahedron, a cube
C_{nh} is C_n with a mirror $\perp$ the axis
C_{nv} is C_n with n mirrors through the axis
D_{nh} is D_n with a mirror $\perp$ the main axis
D_{nd} is D_n with n mirrors between the twofold axes
C_i is inversion alone
C_s is one mirror alone
T_h, O_h are T or O with inversion
T_d is T with diagonal mirrors

Reading a Space-Group Symbol

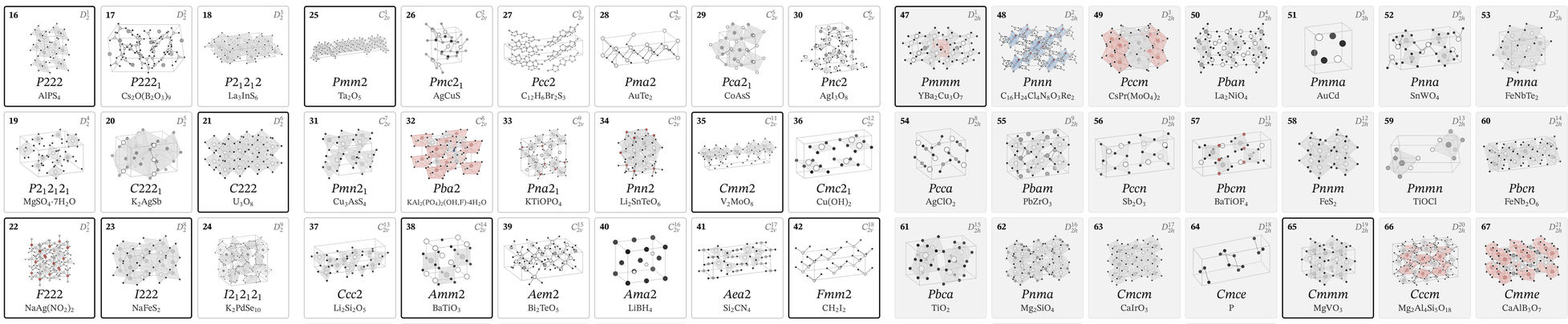


	Monoclinic	Orthorhombic	Tetragonal	Trigonal, Hexagonal	Cubic
Primary	b	a	c	c	⟨100⟩
Secondary		b	a	a	⟨111⟩
Tertiary		c	[110]	[1$\bar{1}$0]	⟨110⟩

Axes: n rotation, $\bar{n}$ rotoinversion, n_k screw. Planes: m mirror; a, b, c, n, d, e glides. The point group reads the same way: drop the lattice letter, make screws rotations and glides mirrors ($P4_2/mnm$ has $4/mmm$).

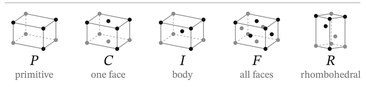
Orthorhombic

$222\ \chi$ $\quad mm2\uparrow$ $\quad mmm\ \bar{\imath}$

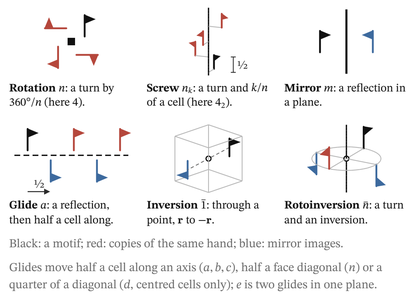


19 Epsom salt
21 U_3O_8 yellowcake
33 $KTiOPO_4$ green lasers
47 YBCO, 93 K superconductor
55 $PbZrO_3$ antiferroelectric
61 TiO_2 brookite
62 Mg_2SiO_4 olivine, upper mantle
64 P black phosphorus
66 Cordierite, catalytic converters
69 $BaFe_2As_2$ iron-pnictide parent
70 S_8 sulfur

Lattice Centrings



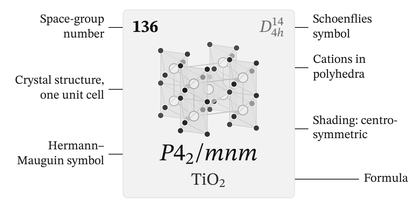
Symmetry Operations



Black: a motif; red: copies of the same hand; blue: mirror images.

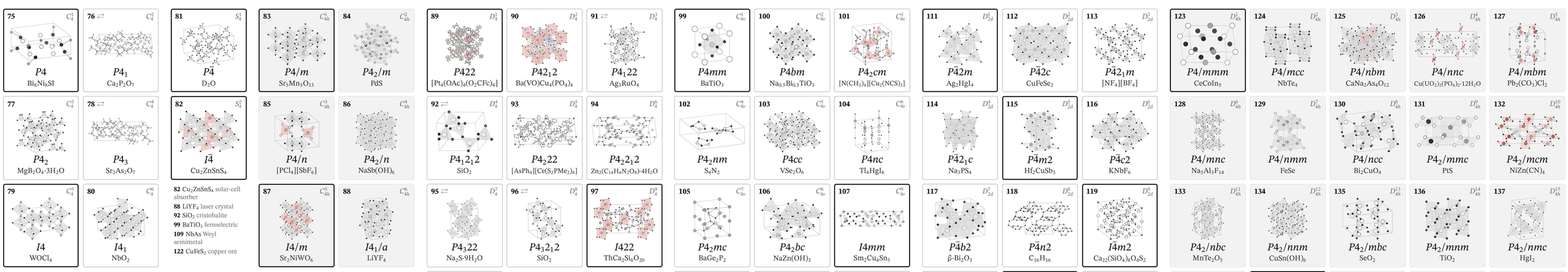
Glides move half a cell along an axis (a, b, c), half a face diagonal (n) or a quarter of a diagonal (d, centred cells only); e is two glides in one plane.

Reading a Tile



Tetragonal

$4\ \chi\uparrow$ $\quad \bar{4}$ $\quad 4/m\ \bar{\imath}$ $\quad 422\ \chi$ $\quad 4mm\uparrow$ $\quad \bar{4}2m$ $\quad 4/mmm\ \bar{\imath}$



82 Cu_2ZnSnS_4 solar-cell absorber
88 LiYF$_4$ laser crystal
92 SiO_2 cristobalite
99 $BaTiO_3$ ferroelectric
109 NbAs Weyl semimetal
122 $CuFeS_2$ copper ore
123 $CeCoIn_5$ heavy-fermion superconductor
129 FeSe iron-based superconductor
136 TiO_2 rutile, white pigment
137 HgI_2 X-ray detector
139 Sr_2RuO_4 unconventional superconductor
140 $CuAl_2$ hardens aluminium alloys
141 $ZrSiO_4$ zircon, oldest minerals
142 Sr_2IrO_4 spin–orbit Mott insulator

Finding the 230

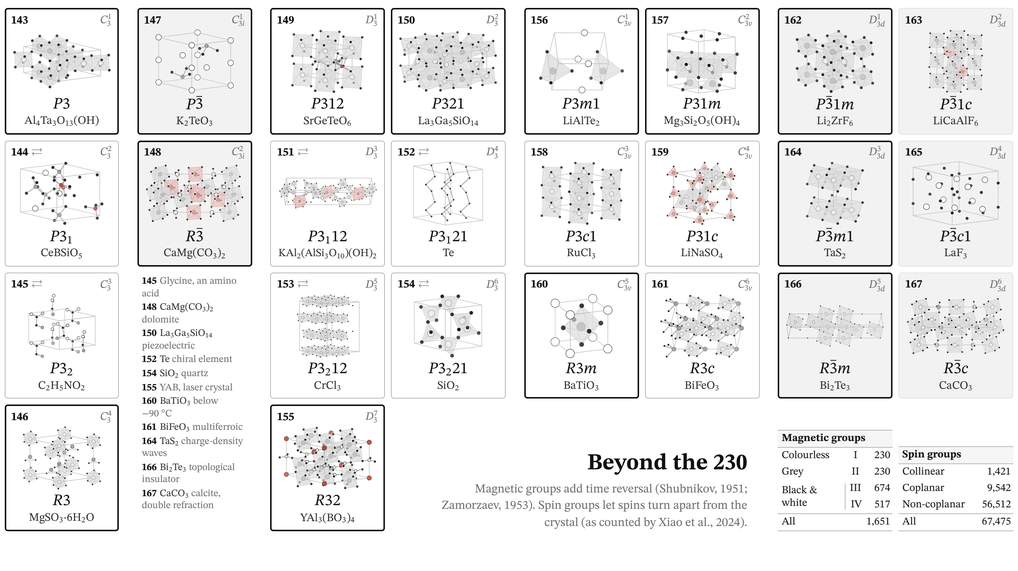


E. Fedorov 1853–1919 · A. Schoenflies 1853–1928 · W. Barlow 1845–1934

Fedorov and Schoenflies each listed the space groups in 1891, both with omissions; by 1892 their letters had settled the 230. Barlow found them again in 1894, by another route.

Trigonal

$3\ \chi\uparrow$ $\quad \bar{3}\ \bar{\imath}$ $\quad 32\ \chi$ $\quad 3m\uparrow$ $\quad \bar{3}m\ \bar{\imath}$



145 Glycine, an amino acid
148 $CaMg(CO_3)_2$ dolomite
150 $La_3Ga_5SiO_{14}$ piezoelectric
152 Te chiral element
154 SiO_2 quartz
155 YAB, laser crystal
160 $BaTiO_3$ below −90 °C
161 $BiFeO_3$ multiferroic
164 TaS_2 charge-density waves
166 Bi_2Te_3 topological insulator
167 $CaCO_3$ calcite, double refraction

Beyond the 230

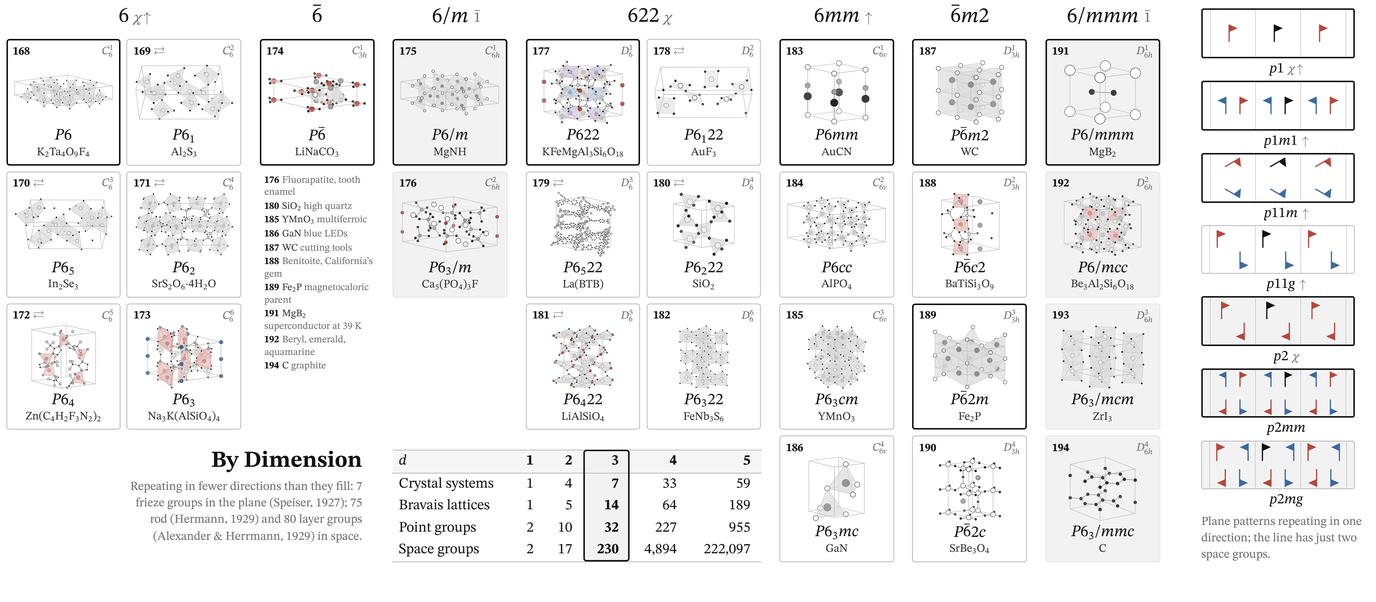
Magnetic groups add time reversal (Shubnikov, 1951; Zamorzaev, 1953). Spin groups let spins turn apart from the crystal (as counted by Xiao et al., 2024).

Magnetic groups		
Colourless	I	230
Grey	II	230
Black & white	III	674
	IV	517
All		1,651

Spin groups	
Collinear	1,421
Coplanar	9,542
Non-coplanar	56,512
All	67,475

Hexagonal

$6\ \chi\uparrow$ $\quad \bar{6}$ $\quad 6/m\ \bar{\imath}$ $\quad 622\ \chi$ $\quad 6mm\uparrow$ $\quad \bar{6}m2$ $\quad 6/mmm\ \bar{\imath}$



169 Al_2S_3
174 Fluorapatite, tooth enamel
180 SiO_2 high quartz
185 $YMnO_3$ multiferroic
186 GaN blue LEDs
187 WC cutting tools
188 Benitoite, California's gem
189 Fe_2P magnetocaloric parent
191 MgB_2 superconductor at 39 K
192 Beryl, emerald, aquamarine
194 C graphite

By Dimension

Repeating in fewer directions than they fill: 7 frieze groups in the plane (Speiser, 1927); 75 rod (Hermann, 1929) and 80 layer groups (Alexander & Herrmann, 1929) in space.

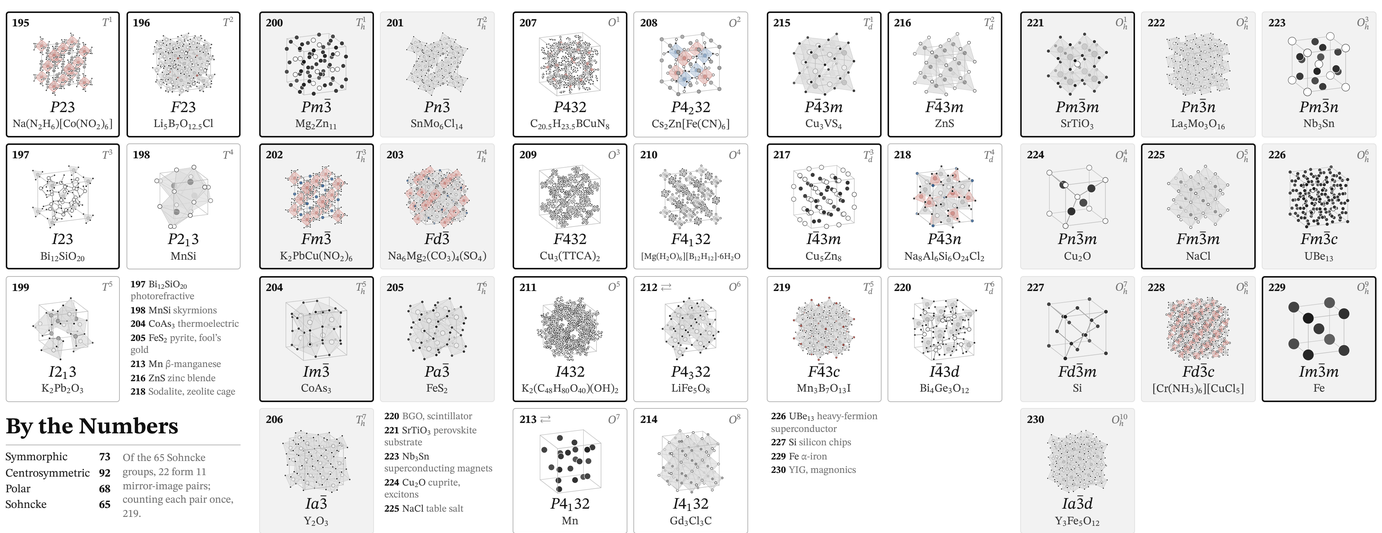
d	1	2	3	4	5
Crystal systems	1	4	7	33	59
Bravais lattices	1	5	14	64	189
Point groups	2	10	32	227	955
Space groups	2	17	230	4,894	222,097

Frieze Groups

Plane patterns repeating in one direction; the line has just two space groups.

Cubic

$23\ \chi$ $\quad m\bar{3}\ \bar{\imath}$ $\quad 432\ \chi$ $\quad \bar{4}3m$ $\quad m\bar{3}m\ \bar{\imath}$

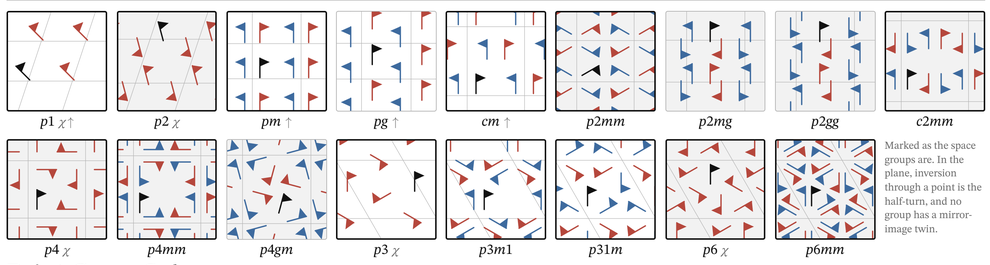


220 BGO, scintillator
221 $SrTiO_3$ perovskite substrate
223 Nb_3Sn superconducting magnets
224 Cu_2O cuprite, excitons
225 NaCl table salt
226 UBe_{13} heavy-fermion superconductor
227 Si silicon chips
229 Fe α-iron
230 YIG, magnonics

By the Numbers

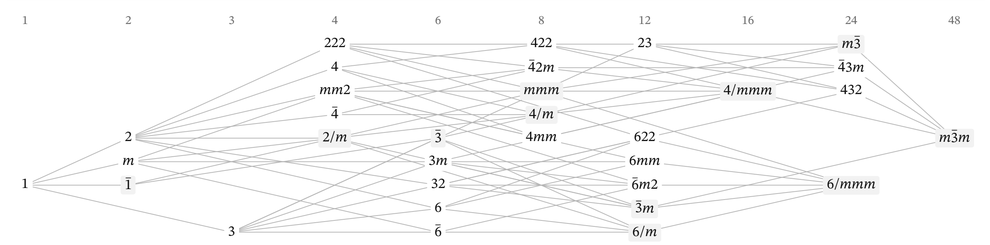
Symmorphic	73	Of the 65 Sohncke groups, 22 form 11 mirror-image pairs; counting each pair once, 219.
Centrosymmetric	92	
Polar	68	
Sohncke	65	

The 17 Wallpaper Groups



Marked as the space groups are. In the plane, inversion through a point is the half-turn, and no group has a mirror-image twin.

Point-Group Subgroups



Left to right by order (the number above each column); a line joins each group to its maximal subgroups. Shaded: centrosymmetric.

Hermann–Mauguin symbols follow the standard settings of International Tables for Crystallography, Vol. A: unique axis b and cell choice 1 for monoclinic groups, origin choice 2, and hexagonal axes for rhombohedral groups. Each crystal is drawn in the cell of its source file; the structures are from the Crystallography Open Database and AFLOW. Inspired by Frank Hoffmann's Space Group List Project. Portraits: Fedorov and Schoenflies, public domain (Wikimedia Commons); Barlow by Walter Stoneman, 1917, © National Portrait Gallery, London (CC BY-NC-ND 3.0).

Interactive edition
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