The 230 Space Groups

Arranged by crystal system and point group, in the order of the International Tables. Each tile pairs a space group with a crystal that has exactly its symmetry.

Key

- $\bar{1}$ — Centrosymmetric: contains the inversion
- χ — Sohncke: proper rotations only
- ↑ — Polar: a direction every operation keeps
- □ — Symmorphic: no screw axes or glide planes
- ⇄ — Enantiomorphic: one of a mirror-image pair

Triclinic

$1\ \chi\uparrow$ $\quad \bar{1}\ \bar{1}$

1 C_1^1 — $P1$ — $Al_2Si_2O_5(OH)_4$

2 C_i^1 — $P\bar{1}$ — $CuSO_4{\cdot}5H_2O$

Monoclinic

$2\ \chi\uparrow$ $\quad m\uparrow$ $\quad 2/m\ \bar{1}$

3 C_2^1 — $P2$ — KYb_2F_7

6 C_s^1 — Pm — $BaBe_2Si_2O_7$

10 C_{2h}^1 — $P2/m$ — $AuAgTe_2$

11 C_{2h}^2 — $P2_1/m$ — $MoTe_2$

4 C_2^2 — $P2_1$ — $C_{12}H_{22}O_{11}$

7 C_s^2 — Pc — $Sn_2P_2S_6$

12 C_{2h}^3 — $C2/m$ — $KAlSi_3O_8$

13 C_{2h}^4 — $P2/c$ — $MnWO_4$

5 C_2^3 — $C2$ — BiB_3O_6

8 C_s^3 — Cm — $Na_2(Sb,As)_8S_{13}{\cdot}2H_2O$

14 C_{2h}^5 — $P2_1/c$ — VO_2

15 C_{2h}^6 — $C2/c$ — $NaAlSi_2O_6$

9 C_s^4 — Cc — $FeMo_2S_4$

↑ Pyroelectricity, ferroelectricity

□ SHG, piezoelectricity (all but 432)

χ Chirality, optical activity

1 Kaolinite, porcelain clay
2 $CuSO_4{\cdot}5H_2O$ copper sulfate
4 $C_{12}H_{22}O_{11}$ table sugar
5 BiB_3O_6 nonlinear optical crystal
11 $MoTe_2$ topological semimetal
12 $KAlSi_3O_8$ orthoclase feldspar
13 $MnWO_4$ multiferroic
14 VO_2 metal–insulator transition
15 $NaAlSi_2O_6$ jade

Crystal Systems

Triclinic — P — no conditions

Monoclinic — P, C — $\alpha = \gamma = 90°$

Orthorhombic — P, C, I, F — $\alpha = \beta = \gamma = 90°$

Tetragonal — P, I — $a = b$; $\alpha = \beta = \gamma = 90°$

Trigonal — (P), R — $a = b$, $\gamma = 120°$; or $a = b = c$, $\alpha = \beta = \gamma$

Hexagonal — P — $a = b$; $\alpha = \beta = 90°$, $\gamma = 120°$

Cubic — P, I, F — $a = b = c$; $\alpha = \beta = \gamma = 90°$

With its centrings, each system gives the 14 Bravais lattices (trigonal's P is hexagonal's).

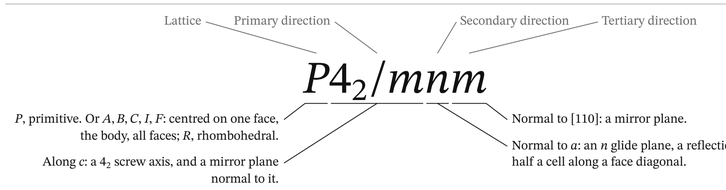
The 32 Point Groups

Triclinic	Monoclinic	Orthorhombic	Tetragonal	Trigonal	Hexagonal	Cubic
1 C_1 1	2 C_2 3	222 D_2 9	4 C_4 6	3 C_3 4	6 C_6 6	23 T 5
$\bar{1}$ C_i 1	m C_s 4	$mm2$ C_{2v} 22	$\bar{4}$ S_4 2	$\bar{3}$ C_{3i} 2	$\bar{6}$ C_{3h} 1	$m\bar{3}$ T_h 7
	$2/m$ C_{2h} 6	mmm D_{2h} 28	$4/m$ C_{4h} 6	32 D_3 7	$6/m$ C_{6h} 2	432 O 8
			422 D_4 10	$3m$ C_{3v} 6	622 D_6 6	$\bar{4}3m$ T_d 6
			$4mm$ C_{4v} 12	$\bar{3}m$ D_{3d} 6	$6mm$ C_{6v} 4	$m\bar{3}m$ O_h 10
			$\bar{4}2m$ D_{2d} 12		$\bar{6}m2$ D_{3h} 4	
			$4/mmm$ D_{4h} 20		$6/mmm$ D_{6h} 4	

Each in Hermann–Mauguin notation, with its number of space groups; shaded: centrosymmetric. First derived by Hessel (1830) and, independently, Gadolin (1867).

C_n is an n-fold rotation axis
D_n is C_n with n twofold axes $\perp$ it
S_n is an n-fold rotoreflection axis
T, O are the rotations of a tetrahedron, a cube
C_{nh} is C_n with a mirror $\perp$ the axis
C_{nv} is C_n with n mirrors through the axis
D_{nh} is D_n with a mirror $\perp$ the main axis
D_{nd} is D_n with n mirrors between the twofold axes
C_i is inversion alone
C_s is one mirror alone
T_h, O_h are T or O with inversion
T_d is T with diagonal mirrors

Reading a Space-Group Symbol



	Monoclinic	Orthorhombic	Tetragonal	Trigonal, Hexagonal	Cubic
Primary	b	a	c	c	⟨100⟩
Secondary		b	a	a	⟨111⟩
Tertiary		c	[110]	$[1\bar{1}0]$	⟨110⟩

Axes: n rotation, $\bar{n}$ rotoinversion, n_k screw. Planes: m mirror; a, b, c, n, d, e glides. The point group reads the same way: drop the lattice letter, make screws rotations and glides mirrors ($P4_2/mnm$ has $4/mmm$).

Orthorhombic

$222\ \chi$ $\quad mm2\uparrow$ $\quad mmm\ \bar{1}$

16 D_2^1 — $P222$ — $AlPS_4$

17 D_2^2 — $P222_1$ — $Cs_2O(B_2O_3)_9$

18 D_2^3 — $P2_12_12$ — La_3InSe_6

25 C_{2v}^1 — $Pmm2$ — Ta_2O_5

26 C_{2v}^2 — $Pmc2_1$ — $AgCuS$

27 C_{2v}^3 — $Pcc2$ — $C_{12}H_6Br_2S_3$

28 C_{2v}^4 — $Pma2$ — $AuTe_2$

29 C_{2v}^5 — $Pca2_1$ — $CoAsS$

30 C_{2v}^6 — $Pnc2$ — AgI_3O_8

47 D_{2h}^1 — $Pmmm$ — $YBa_2Cu_3O_7$

48 D_{2h}^2 — $Pnnn$ — $C_{16}H_{24}Cl_4N_8O_3Re_2$

49 D_{2h}^3 — $Pccm$ — $CsPr(MoO_4)_2$

50 D_{2h}^4 — $Pban$ — La_2NiO_4

51 D_{2h}^5 — $Pmma$ — $AuCd$

52 D_{2h}^6 — $Pnna$ — $SnWO_4$

53 D_{2h}^7 — $Pmna$ — $FeNbTe_2$

19 D_2^4 — $P2_12_12_1$ — $MgSO_4{\cdot}7H_2O$

20 D_2^5 — $C222_1$ — K_2AgSb

21 D_2^6 — $C222$ — U_3O_8

31 C_{2v}^7 — $Pmn2_1$ — Cu_3AsS_4

32 C_{2v}^8 — $Pba2$ — $KAl_2(PO_4)_2(OH,F){\cdot}4H_2O$

33 C_{2v}^9 — $Pna2_1$ — $KTiOPO_4$

34 C_{2v}^{10} — $Pnn2$ — Li_2SnTeO_6

35 C_{2v}^{11} — $Cmm2$ — V_2MoO_8

36 C_{2v}^{12} — $Cmc2_1$ — $Cu(OH)_2$

54 D_{2h}^8 — $Pcca$ — $AgClO_2$

55 D_{2h}^9 — $Pbam$ — $PbZrO_3$

56 D_{2h}^{10} — $Pccn$ — Sb_2O_3

57 D_{2h}^{11} — $Pbcm$ — $BaTiOF_4$

58 D_{2h}^{12} — $Pnnm$ — FeS_2

59 D_{2h}^{13} — $Pmmn$ — $TiOCl$

60 D_{2h}^{14} — $Pbcn$ — $FeNb_2O_6$

22 D_2^7 — $F222$ — $NaAg(NO_2)_2$

23 D_2^8 — $I222$ — $NaFeS_2$

24 D_2^9 — $I2_12_12_1$ — K_2PdSe_{10}

37 C_{2v}^{13} — $Ccc2$ — $Li_2Si_2O_5$

38 C_{2v}^{14} — $Amm2$ — $BaTiO_3$

39 C_{2v}^{15} — $Aem2$ — Bi_2TeO_5

40 C_{2v}^{16} — $Ama2$ — $LiBH_4$

41 C_{2v}^{17} — $Aea2$ — Si_2CN_4

42 C_{2v}^{18} — $Fmm2$ — CH_2I_2

61 D_{2h}^{15} — $Pbca$ — TiO_2

62 D_{2h}^{16} — $Pnma$ — Mg_2SiO_4

63 D_{2h}^{17} — $Cmcm$ — $CaIrO_3$

64 D_{2h}^{18} — $Cmce$ — P

65 D_{2h}^{19} — $Cmmm$ — $MgVO_3$

66 D_{2h}^{20} — $Cccm$ — $Mg_2Al_4Si_5O_{18}$

67 D_{2h}^{21} — $Cmme$ — $CaAlB_3O_7$

43 C_{2v}^{19} — $Fdd2$ — $TlPbAs_3S_6$

44 C_{2v}^{20} — $Imm2$ — $AgNO_2$

45 C_{2v}^{21} — $Iba2$ — $BaNa_2Al_4Si_4O_{16}$

46 C_{2v}^{22} — $Ima2$ — $Na_2BaTi_2O_2[Si_4O_{12}]$

68 D_{2h}^{22} — $Ccce$ — $Ba(H_2PO_2)_2$

69 D_{2h}^{23} — $Fmmm$ — $BaFe_2As_2$

70 D_{2h}^{24} — $Fddd$ — S_8

71 D_{2h}^{25} — $Immm$ — Ni_2V

72 D_{2h}^{26} — $Ibam$ — $PbCu_3(VO_4)_2Cl_2$

73 D_{2h}^{27} — $Ibca$ — $Na_2[SiO_2(OH)_2]{\cdot}8H_2O$

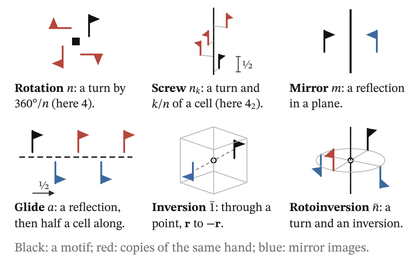
74 D_{2h}^{28} — $Imma$ — Na_2MgAlF_7

19 Epsom salt
21 U_3O_8 yellowcake
33 $KTiOPO_4$ green lasers
47 YBCO, 93 K superconductor
55 $PbZrO_3$ antiferroelectric
61 TiO_2 brookite
62 Mg_2SiO_4 olivine, upper mantle
64 P black phosphorus
66 Cordierite, catalytic converters
69 $BaFe_2As_2$ iron-pnictide parent
70 S_8 sulfur

Lattice Centrings

P primitive — C one face — I body — F all faces — R rhombohedral

Symmetry Operations

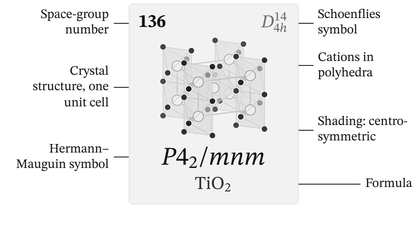


Rotation n: a turn by $360°/n$ (here 4).
Screw n_k: a turn and k/n of a cell (here 4_2).
Mirror m: a reflection in a plane.
Glide a: a reflection, then half a cell along.
Inversion $\bar{1}$: through a point, $\mathbf{r}$ to $-\mathbf{r}$.
Rotoinversion $\bar{n}$: a turn and an inversion.

Black: a motif; red: copies of the same hand; blue: mirror images.

Glides move half a cell along an axis (a, b, c), half a face diagonal (n) or a quarter of a diagonal (d, centred cells only); e is two glides in one plane.

Reading a Tile



Tetragonal

$4\ \chi\uparrow$ $\quad \bar{4}$ $\quad 4/m\ \bar{1}$ $\quad 422\ \chi$ $\quad 4mm\uparrow$ $\quad \bar{4}2m$ $\quad 4/mmm\ \bar{1}$

75 C_4^1 — $P4$ — Bi_8Ni_8SI

76 ⇄ C_4^2 — $P4_1$ — $Ca_2P_2O_7$

81 S_4^1 — $P\bar{4}$ — D_2O

83 C_{4h}^1 — $P4/m$ — $Sr_5Mn_5O_{13}$

84 C_{4h}^2 — $P4_2/m$ — PdS

89 D_4^1 — $P422$ — $[Pt_4(OAc)_4(O_2CFc)_4]$

90 D_4^2 — $P42_12$ — $Ba(VO)Cu_4(PO_4)_4$

91 ⇄ D_4^3 — $P4_122$ — Ag_3RuO_4

99 C_{4v}^1 — $P4mm$ — $BaTiO_3$

100 C_{4v}^2 — $P4bm$ — $Na_{0.5}Bi_{0.5}TiO_3$

101 C_{4v}^3 — $P4_2cm$ — $[N(CH_3)_4][Cu_3(NCS)_5]$

111 D_{2d}^1 — $P\bar{4}2m$ — Ag_2HgI_4

112 D_{2d}^2 — $P\bar{4}2c$ — $CuFeSe_2$

113 D_{2d}^3 — $P\bar{4}2_1m$ — $[NF_4][BF_4]$

123 D_{4h}^1 — $P4/mmm$ — $CeCoIn_5$

124 D_{4h}^2 — $P4/mcc$ — $NbTe_4$

125 D_{4h}^3 — $P4/nbm$ — $CaNa_2As_4O_{12}$

126 D_{4h}^4 — $P4/nnc$ — $Cu(UO_2)_2(PO_4)_2{\cdot}12H_2O$

127 D_{4h}^5 — $P4/mbm$ — $Pb_2(CO_3)Cl_2$

77 C_4^3 — $P4_2$ — $MgB_2O_4{\cdot}3H_2O$

78 ⇄ C_4^4 — $P4_3$ — $Sr_2As_2O_7$

82 S_4^2 — $I\bar{4}$ — Cu_2ZnSnS_4

85 C_{4h}^3 — $P4/n$ — $[PCl_4][SbF_6]$

86 C_{4h}^4 — $P4_2/n$ — $NaSb(OH)_6$

92 ⇄ D_4^4 — $P4_12_12$ — SiO_2

93 D_4^5 — $P4_222$ — $[AsPh_4][Ce(S_2PMe_2)_4]$

94 D_4^6 — $P4_22_12$ — $Zn_2(C_{14}H_4N_2O_8){\cdot}4H_2O$

102 C_{4v}^4 — $P4_2nm$ — S_4N_2

103 C_{4v}^5 — $P4cc$ — VSe_2O_6

104 C_{4v}^6 — $P4nc$ — Tl_4HgI_6

114 D_{2d}^4 — $P\bar{4}2_1c$ — Na_3PS_4

115 D_{2d}^5 — $P\bar{4}m2$ — Hf_2CuSb_3

116 D_{2d}^6 — $P\bar{4}c2$ — $KNbF_6$

128 D_{4h}^6 — $P4/mnc$ — $Na_5Al_3F_{14}$

129 D_{4h}^7 — $P4/nmm$ — FeSe

130 D_{4h}^8 — $P4/ncc$ — Bi_2CuO_4

131 D_{4h}^9 — $P4_2/mmc$ — PtS

132 D_{4h}^{10} — $P4_2/mcm$ — NiZn(CN)$_4$

79 C_4^5 — $I4$ — $WOCl_4$

80 C_4^6 — $I4_1$ — NbO_2

87 C_{4h}^5 — $I4/m$ — Sr_2NiWO_6

88 C_{4h}^6 — $I4_1/a$ — $LiYF_4$

95 ⇄ D_4^7 — $P4_322$ — $Na_2S{\cdot}9H_2O$

96 ⇄ D_4^8 — $P4_32_12$ — SiO_2

97 D_4^9 — $I422$ — $ThCa_2Si_8O_{20}$

98 D_4^{10} — $I4_122$ — $C_8H_5MgO_5$

105 C_{4v}^7 — $P4_2mc$ — $BaGe_2P_2$

106 C_{4v}^8 — $P4_2bc$ — $NaZn(OH)_3$

107 C_{4v}^9 — $I4mm$ — $Sm_2Cu_4Sn_5$

117 D_{2d}^7 — $P\bar{4}b2$ — β-Bi_2O_3

118 D_{2d}^8 — $P\bar{4}n2$ — $C_{16}H_{16}$

119 D_{2d}^9 — $I\bar{4}m2$ — $Ca_{22}(SiO_4)_8O_4S_2$

133 D_{4h}^{11} — $P4_2/nbc$ — $MnTe_2O_5$

134 D_{4h}^{12} — $P4_2/nnm$ — $CuSn(OH)_6$

135 D_{4h}^{13} — $P4_2/mbc$ — SeO_2

136 D_{4h}^{14} — $P4_2/mnm$ — TiO_2

137 D_{4h}^{15} — $P4_2/nmc$ — HgI_2

108 C_{4v}^{10} — $I4cm$ — $Rb_5Nb_3OF_{18}$

109 C_{4v}^{11} — $I4_1md$ — NbAs

110 C_{4v}^{12} — $I4_1cd$ — $Be(BH_4)_2$

120 D_{2d}^{10} — $I\bar{4}c2$ — $BeSO_4{\cdot}4H_2O$

121 D_{2d}^{11} — $I\bar{4}2m$ — Cu_2FeSnS_4

122 D_{2d}^{12} — $I\bar{4}2d$ — $CuFeS_2$

138 D_{4h}^{16} — $P4_2/ncm$ — AuI

139 D_{4h}^{17} — $I4/mmm$ — Sr_2RuO_4

140 D_{4h}^{18} — $I4/mcm$ — $CuAl_2$

141 D_{4h}^{19} — $I4_1/amd$ — $ZrSiO_4$

142 D_{4h}^{20} — $I4_1/acd$ — Sr_2IrO_4

82 Cu_2ZnSnS_4 solar-cell absorber
88 $LiYF_4$ laser crystal
92 SiO_2 cristobalite
99 $BaTiO_3$ ferroelectric
109 NbAs Weyl semimetal
122 $CuFeS_2$ copper ore
123 $CeCoIn_5$ heavy-fermion superconductor
129 FeSe iron-based superconductor
136 TiO_2 rutile, white pigment
137 HgI_2 X-ray detector
139 Sr_2RuO_4 unconventional superconductor
140 $CuAl_2$ hardens aluminium alloys
141 $ZrSiO_4$ zircon, oldest minerals
142 Sr_2IrO_4 spin–orbit Mott insulator

Finding the 230



E. Fedorov 1853–1919 — A. Schoenflies 1853–1928 — W. Barlow 1845–1934

Fedorov and Schoenflies each listed the space groups in 1891, both with omissions; by 1892 their letters had settled the 230. Barlow found them again in 1894, by another route.

Trigonal

$3\ \chi\uparrow$ $\quad \bar{3}\ \bar{1}$ $\quad 32\ \chi$ $\quad 3m\uparrow$ $\quad \bar{3}m\ \bar{1}$

143 C_3^1 — $P3$ — $Al_4Ta_3O_{13}(OH)$

147 C_{3i}^1 — $P\bar{3}$ — K_2TeO_3

149 D_3^1 — $P312$ — $SrGeTeO_6$

150 D_3^2 — $P321$ — $La_3Ga_5SiO_{14}$

156 C_{3v}^1 — $P3m1$ — $LiAlTe_2$

157 C_{3v}^2 — $P31m$ — $Mg_3Si_2O_5(OH)_4$

162 D_{3d}^1 — $P\bar{3}1m$ — Li_2ZrF_6

163 D_{3d}^2 — $P\bar{3}1c$ — $LiCaAlF_6$

144 ⇄ C_3^2 — $P3_1$ — CeBSiO$_5$

148 C_{3i}^2 — $R\bar{3}$ — $CaMg(CO_3)_2$

151 ⇄ D_3^3 — $P3_112$ — $KAl_2(AlSi_3O_{10})(OH)_2$

152 ⇄ D_3^4 — $P3_121$ — Te

158 C_{3v}^3 — $P3c1$ — $RuCl_3$

159 C_{3v}^4 — $P31c$ — $LiNaSO_4$

164 D_{3d}^3 — $P\bar{3}m1$ — TaS_2

165 D_{3d}^4 — $P\bar{3}c1$ — LaF_3

145 ⇄ C_3^3 — $P3_2$ — $C_2H_5NO_2$

153 ⇄ D_3^5 — $P3_212$ — $CrCl_3$

154 ⇄ D_3^6 — $P3_221$ — SiO_2

160 C_{3v}^5 — $R3m$ — $BaTiO_3$

161 C_{3v}^6 — $R3c$ — $BiFeO_3$

166 D_{3d}^5 — $R\bar{3}m$ — Bi_2Te_3

167 D_{3d}^6 — $R\bar{3}c$ — $CaCO_3$

146 C_3^4 — $R3$ — $MgSO_3{\cdot}6H_2O$

155 D_3^7 — $R32$ — $YAl_3(BO_3)_4$

145 Glycine, an amino acid
148 $CaMg(CO_3)_2$ dolomite
150 $La_3Ga_5SiO_{14}$ piezoelectric
152 Te chiral element
154 SiO_2 quartz
155 YAB, laser crystal
160 $BaTiO_3$ below −90 °C
161 $BiFeO_3$ multiferroic
164 TaS_2 charge-density waves
166 Bi_2Te_3 topological insulator
167 $CaCO_3$ calcite, double refraction

Beyond the 230

Magnetic groups add time reversal (Shubnikov, 1951; Zamorzaev, 1953). Spin groups let spins turn apart from the crystal (as counted by Xiao et al., 2024).

Magnetic groups		
Colourless	I	230
Grey	II	230
Black & white	III	674
	IV	517
All		1,651

Spin groups	
Collinear	1,421
Coplanar	9,542
Non-coplanar	56,512
All	67,475

Hexagonal

$6\ \chi\uparrow$ $\quad \bar{6}$ $\quad 6/m\ \bar{1}$ $\quad 622\ \chi$ $\quad 6mm\uparrow$ $\quad \bar{6}m2$ $\quad 6/mmm\ \bar{1}$

168 C_6^1 — $P6$ — $K_2Ta_4O_9F_4$

169 ⇄ C_6^2 — $P6_1$ — Al_2S_3

174 C_{3h}^1 — $P\bar{6}$ — $LiNaCO_3$

175 C_{6h}^1 — $P6/m$ — MgNH

177 D_6^1 — $P622$ — $KFeMgAl_3Si_6O_{18}$

178 ⇄ D_6^2 — $P6_122$ — AuF_3

183 C_{6v}^1 — $P6mm$ — AuCN

187 D_{3h}^1 — $P\bar{6}m2$ — WC

191 D_{6h}^1 — $P6/mmm$ — MgB_2

170 ⇄ C_6^3 — $P6_5$ — In_2Se_3

171 ⇄ C_6^4 — $P6_2$ — $SrS_2O_6{\cdot}4H_2O$

176 Fluorapatite, tooth enamel
180 SiO_2 high quartz
185 $YMnO_3$ multiferroic
186 GaN blue LEDs
187 WC cutting tools
188 Benitoite, California's gem
189 Fe_2P magnetocaloric parent
191 MgB_2 superconductor at 39 K
192 Beryl, emerald, aquamarine
194 C graphite

176 C_{6h}^2 — $P6_3/m$ — $Ca_5(PO_4)_3F$

179 ⇄ D_6^3 — $P6_522$ — La(BTB)

180 ⇄ D_6^4 — $P6_222$ — SiO_2

184 C_{6v}^2 — $P6cc$ — $AlPO_4$

188 D_{3h}^2 — $P\bar{6}c2$ — $BaTiSi_3O_9$

192 D_{6h}^2 — $P6/mcc$ — $Be_3Al_2Si_6O_{18}$

172 ⇄ C_6^5 — $P6_4$ — $Zn(C_4H_2F_3N_2)_2$

173 C_6^6 — $P6_3$ — $Na_3K(AlSiO_4)_4$

181 ⇄ D_6^5 — $P6_422$ — $LiAlSiO_4$

182 D_6^6 — $P6_322$ — $FeNb_3S_6$

185 C_{6v}^3 — $P6_3cm$ — $YMnO_3$

189 D_{3h}^3 — $P\bar{6}2m$ — Fe_2P

193 D_{6h}^3 — $P6_3/mcm$ — ZrI_3

186 C_{6v}^4 — $P6_3mc$ — GaN

190 D_{3h}^4 — $P\bar{6}2c$ — $SrBe_3O_4$

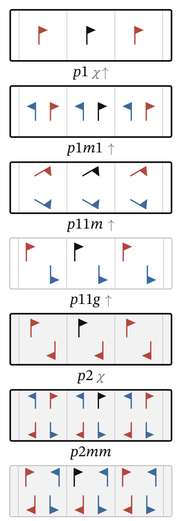
194 D_{6h}^4 — $P6_3/mmc$ — C

By Dimension

Repeating in fewer directions than they fill: 7 frieze groups in the plane (Speiser, 1927); 75 rod (Hermann, 1929) and 80 layer groups (Alexander & Herrmann, 1929) in space.

d	1	2	**3**	4	5
Crystal systems	1	4	**7**	33	59
Bravais lattices	1	5	**14**	64	189
Point groups	2	10	**32**	227	955
Space groups	2	17	**230**	4,894	222,097

Frieze Groups



Plane patterns repeating in one direction; the line has just two space groups.

Cubic

$23\ \chi$ $\quad m\bar{3}\ \bar{1}$ $\quad 432\ \chi$ $\quad \bar{4}3m$ $\quad m\bar{3}m\ \bar{1}$

195 T^1 — $P23$ — $Na(N_2H_5)[Co(NO_2)_6]$

196 T^2 — $F23$ — $Li_5B_7O_{12.5}Cl$

200 T_h^1 — $Pm\bar{3}$ — Mg_2Zn_{11}

201 T_h^2 — $Pn\bar{3}$ — $SnMo_6Cl_{14}$

207 O^1 — $P432$ — $C_{20.5}H_{23.5}BCuN_8$

208 O^2 — $P4_232$ — $Cs_2Zn[Fe(CN)_6]$

215 T_d^1 — $P\bar{4}3m$ — Cu_3VS_4

216 T_d^2 — $F\bar{4}3m$ — ZnS

221 O_h^1 — $Pm\bar{3}m$ — $SrTiO_3$

222 O_h^2 — $Pn\bar{3}n$ — $La_5Mo_3O_{16}$

223 O_h^3 — $Pm\bar{3}n$ — Nb_3Sn

197 T^3 — $I23$ — $Bi_{12}SiO_{20}$

198 T^4 — $P2_13$ — MnSi

202 T_h^3 — $Fm\bar{3}$ — $K_2PbCu(NO_2)_6$

203 T_h^4 — $Fd\bar{3}$ — $Na_6Mg_2(CO_3)_4(SO_4)$

209 O^3 — $F432$ — $Cu_3(TTCA)_2$

210 O^4 — $F4_132$ — $[Mg(H_2O)_6][B_{12}H_{12}]{\cdot}6H_2O$

217 T_d^3 — $I\bar{4}3m$ — Cu_5Zn_8

218 T_d^4 — $P\bar{4}3n$ — $Na_8Al_6Si_6O_{24}Cl_2$

224 O_h^4 — $Pn\bar{3}m$ — Cu_2O

225 O_h^5 — $Fm\bar{3}m$ — NaCl

226 O_h^6 — $Fm\bar{3}c$ — UBe_{13}

199 T^5 — $I2_13$ — $K_2Pb_2O_3$

204 T_h^5 — $Im\bar{3}$ — $CoAs_3$

205 T_h^6 — $Pa\bar{3}$ — FeS_2

211 O^5 — $I432$ — $K_2(C_{48}H_{80}O_{40})(OH)_2$

212 ⇄ O^6 — $P4_332$ — $LiFe_5O_8$

219 T_d^5 — $F\bar{4}3c$ — $Mn_3B_7O_{13}I$

220 T_d^6 — $I\bar{4}3d$ — $Bi_4Ge_3O_{12}$

227 O_h^7 — $Fd\bar{3}m$ — Si

228 O_h^8 — $Fd\bar{3}c$ — $[Cr(NH_3)_6][CuCl_5]$

229 O_h^9 — $Im\bar{3}m$ — Fe

206 T_h^7 — $Ia\bar{3}$ — Y_2O_3

213 ⇄ O^7 — $P4_132$ — Mn

214 O^8 — $I4_132$ — Gd_3Cl_3C

230 O_h^{10} — $Ia\bar{3}d$ — $Y_3Fe_5O_{12}$

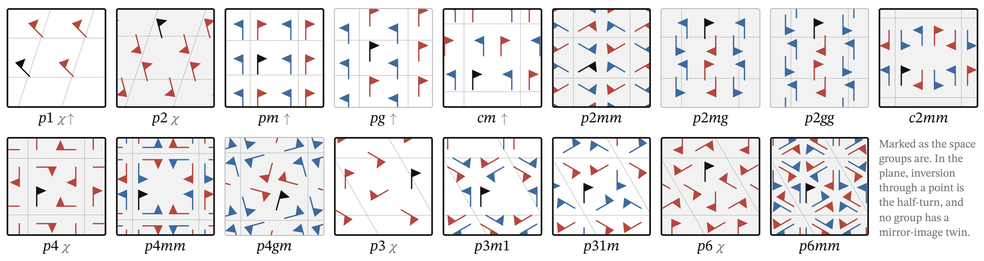
197 $Bi_{12}SiO_{20}$ photorefractive
198 MnSi skyrmions
204 $CoAs_3$ thermoelectric
205 FeS_2 pyrite, fool's gold
213 Mn β-manganese
216 ZnS zinc blende
218 Sodalite, zeolite cage
220 BGO, scintillator
221 $SrTiO_3$ perovskite substrate
223 Nb_3Sn superconducting magnets
224 Cu_2O cuprite, excitons
225 NaCl table salt
226 UBe_{13} heavy-fermion superconductor
227 Si silicon chips
229 Fe α-iron
230 YIG, magnonics

By the Numbers

Symmorphic	73
Centrosymmetric	92
Polar	68
Sohncke	65

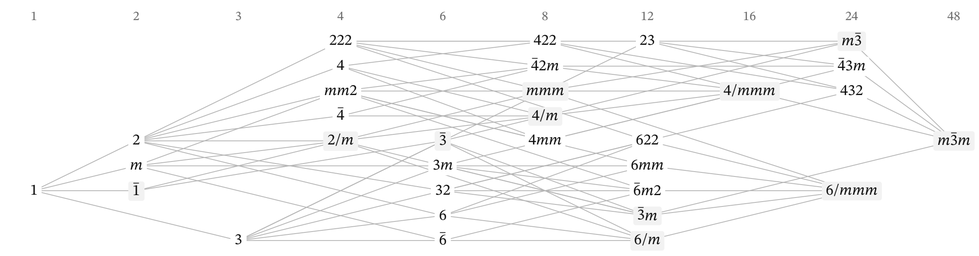
Of the 65 Sohncke groups, 22 form 11 mirror-image pairs; counting each pair once, 219.

The 17 Wallpaper Groups



Marked as the space groups are. In the plane, inversion through a point is the half-turn, and no group has a mirror-image twin.

Point-Group Subgroups



Left to right by order (the number above each column); a line joins each group to its maximal subgroups. Shaded: centrosymmetric.

Hermann–Mauguin symbols follow the standard settings of International Tables for Crystallography, Vol. A: unique axis b and cell choice 1 for monoclinic groups, origin choice 2, and hexagonal axes for rhombohedral groups. Each crystal is drawn in the cell of its source file; the structures are from the Crystallography Open Database and AFLOW. Inspired by Frank Hoffmann's Space Group List Project. Portraits: Fedorov and Schoenflies, public domain (Wikimedia Commons); Barlow by Walter Stoneman, 1917, © National Portrait Gallery, London (CC BY-NC-ND 3.0).

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